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OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. XXXVIII. PART IV.—FOR THE SESSION 1895–96.

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XXII.—*Observations on the Phonograph.* By JOHN G. M'KENDRICK, M.D., Professor of Physiology in the University of Glasgow. (With Two Plates.)

(Read 17th February 1896.)

INTRODUCTION.

1. Since I had the honour of showing the phonograph to the Royal Society of Edinburgh, at a special meeting in November 1894, the instrument has occupied a good deal of my time and attention, and I now venture to give the general results of the investigation.

2. The instrument chiefly studied has been the machine used in this country known as the "Commercial Phonograph." Any records taken by myself have been obtained with the ordinary apparatus forming part of the "commercial" speaker arm, but I have always reproduced these with the aid of the so-called "musical" arm. The commercial machine, or, to give it a better name, the English model, is so geared that the wax cylinder, $6\frac{7}{8}$ inch (197 mm.) in circumference, makes two revolutions in one second, while the spiral grooves described on the cylinder are $\frac{1}{200}$ inch ($\frac{1}{8}$ mm.) apart. A spiral line about 136 yards in length may be described on the cylinder, and the recording or reproducing point travels over this distance in about six minutes.

3. I have also used the American model, which resembles in all essential particulars the one just described, except that the grooves on the cylinder are $\frac{1}{100}$ inch ($\frac{1}{4}$ mm.) instead of $\frac{1}{200}$ ($\frac{1}{8}$ mm.).

I. RESONATORS FOR INCREASING VOLUME OF TONE.

4. By using conical resonators of considerable size, made of thin block tin, the tone of the phonograph can be increased in volume, so that the sounds become audible and agreeable in a large room. The use of resonators is common in America, but these are of comparatively small dimensions, and, while they do not yield the volume of tone obtained by those of greater size, they do not get rid of the upper partials that give the peculiar character to the sounds emitted by the glass disk of the phonograph. To this I attribute the preference often given by phonograph operators to the method of conveying the sounds directly to the ears by flexible tubes. The latter plan no doubt gives volume of tone and also faithfulness of quality, as it carries the sound to within a few millimetres of the drum head, but the advantage is more than compensated by the discomfort of the proceeding, and by the jarring friction noises that are obtruded with painful distinctness. The largest resonator I use is of conical form, about 8 feet

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11 inches in length, with a diameter at its wide end of 3 feet, and at its narrow end of $\frac{3}{4}$ inch. The best results have been obtained with tin resonators. One large wooden resonator, shaped like a four-sided pyramid, formed of wood used for making wooden organ pipes, 10 feet $7\frac{1}{2}$ inches in length, each side being 3 feet in length at the base and $\frac{3}{4}$ inches at the apex, gave excellent results as regards volume and quality, but it had a curious effect of damping or muffling the tones. Resonators of vulcanite, papier-maché were unsatisfactory. A conical resonator about 3 feet in length, 4 inches in diameter at the wide end, and $\frac{3}{4}$ inch at the narrow end, made of thin aluminium, gave a remarkably clear ringing tone. Large resonators appear not only to give roundness and volume to the tone, but also to quench many of the upper partials which cause the hissing noises heard near the glass disk of the phonograph. In this way quality of tone is improved. I have endeavoured to modify the form of the resonators in various ways in the hope of altering the quality of the tone, but without success.

The resonators I employ have the following measurements :—

No.	Material.	Length.	Diameter at wide end.	Diameter at narrow end.	Name.
1.	Tin, . . .	8 feet 11 inches	3 feet	$\frac{3}{4}$ inch	Great resonator.
2.	Tin, . . .	6 feet 6 inches	18 inches	$\frac{3}{4}$ inch	Small resonator.
3.	Wood $\frac{5}{8}$ inch thick,	10 feet $7\frac{1}{2}$ inches	3 feet	$\frac{3}{4}$ inch	Wooden.
4.	Brass, . . .	3 feet $11\frac{3}{4}$ inches	$9\frac{3}{4}$ inches	$\frac{3}{4}$ inch	Brass.
5.	Aluminium, . .	3 feet 1 inch	$7\frac{3}{4}$ inches	$\frac{3}{4}$ inch	Aluminium.

5. I have not been able to find any clear explanation of the mode of action of conical resonators. Experience has shown that a cone is the best form both for a receiver by which sound waves are transmitted to the phonograph disk, and for strengthening the tone in reproduction. When used as a receiver in taking records, the cone should be long and narrow, and, on the other hand, in reproduction, the best results are obtained by increasing the length, and by increasing the diameter at the broad end. Lord RAYLEIGH states that if the diameter of the large open end be small in comparison with the wave length, the waves on arrival suffer reflexion, but by sufficiently prolonging the cone, this reflexion will be diminished, and it will cease when the diameter of the open end includes a large number of wave lengths. Further, he states that, apart from friction, by diminishing the diameter of the narrow end, and at the same time lengthening the cone, it would be possible to obtain from any given source of sound any desired amount of energy, and to transfer this energy from the tube to the surrounding air. This statement suggests the use of conical resonators much larger than those I have yet used. I have observed that the tone is louder when the ear is placed in the axis of the cone than when it receives the sound waves near the side, as if there were a core of greater disturbance passing down the centre of the cone.

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II. The Use of Mr Alfred Graham's Apparatus along with Large Resonators.*

6. If we suspend Mr Graham's variable resistance apparatus over the phonograph and connect it with the receiver, in front of which a large resonator is placed, and if an electric circuit is established, the volume of tone is much increased. I usually employ a dry cell battery composed of three Obach's cells. Each cell = 2 volts, and with the resistance of the variable resistance and of the receiver gives a current of about $\frac{1}{3}$ of an ampere. It must be admitted, however, that the quality of tone is altered, sometimes apparently for the better, but at other times for the worse. Thus band records giving complex sounds are often harsh, and the human voice often loses in distinctness of articulation. On the other hand, simpler sounds, such as those of the cornet, bass-tuba, saxophone, and bassoon, are sometimes richer and fuller and more like the tone of the real instrument than when the sounds are strengthened by resonance alone. I have observed that by the electrical method a gain beyond a certain limit of loudness is at the expense of clearness and quality. If one is satisfied with the loudness obtained by using one cell instead of three or four, good quality and distinctness are obtained. This is a point of considerable interest, as it shows that the most complex sound waves can be transmuted by the variable resistance apparatus into electrical waves which, in turn, in the receiving apparatus, cause the ferrotype plate of the latter so to vibrate as to give out tones closely resembling those that originally fell on the glass disk of the phonograph.

III. The Use of a Parabolic Reflector.

7. Reflexion of waves of sound from parabolic surfaces is a well-known phenomenon. Rays of sound diverging from the focus and falling on a paraboloid formed by the revolution of a parabola about its axis will be reflected in directions parallel to the axis. I had a paraboloid constructed of zinc, 40 inches in diameter at the open end, and having its focus 5 inches from the vertex. When the wide end of the brass resonator No. 4 in table is placed in the focus, and the narrow end is connected with the tube leading to the phonograph disk, the sound is reflected with a pleasing effect, owing to the friction noises of the phonograph becoming much less audible than when they escape from the wide end of the resonators Nos. 1 and 2.

IV. The taking of Records.

8. Considerable experience has convinced me that the phonograph will record and faithfully reproduce all manner of sounds, provided these are of sufficient intensity to cause the glass disk to vibrate. When I have failed in obtaining a record, investigation

* I have described this apparatus in *Proceedings Royal Society, Edinburgh*, 1896, p. 47.

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has always shown that the fault does not rest so much with the instrument as with the operator. As examples of complex sounds, I have obtained the sound of about 2000 persons singing in a large hall with organ accompaniment, the dashing of the waves on a shingle beach, the hissing produced by the escape of air from an iron cylinder under a pressure of over 200 atmospheres, the sound of a riveter's workshop, and the sound of a military band of about seventy performers. The tones of single instruments are, as a rule, easily taken, and their range as regards pitch is faithfully recorded. Thus, I have records of scales from the deepest tone of the bass-tuba, 28 vibrations per second (the lowest sound in the orchestra except the still lower note of the contra-bassoon, 27 vibrations per second), up to the highest note of the piccolo, 4096 vibrations per second. My collection now includes, taking the instruments in the order of their highest notes, the organ, piccolo, clarinette, harp, flute, piano, violin, guitar, viol d'amour, oboe, saxophone, mandolin, bugle, three-keyed horn, trumpet, cornet, viola, cor anglais, violoncello, alto trombone, tenor trombone, bassoon, bass-tuba, bass trombone. The instruments most difficult to record are those having a wide keyboard, such as the organ, harmonium, and piano. When the sounds proceeding from a series of wires or reeds or pipes stretching over a distance considerably greater than the transverse diameter of the receiving resonator fall on the glass disk of the phonograph, the sounds are not recorded with equal intensity, those coming from the vibrating body immediately in front of the resonator being usually more intense than those coming from vibrating bodies placed more to the sides. This is especially the case when the wide end of the resonator is brought close to the piano, as I have proved by placing the phonograph below the piano and then taking the vibrations from the sounding board immediately overhead. As a rule, both with the organ and piano, the deep pedal notes are not recorded distinctly, and produce jarring sounds probably by the vibrations acting on the loose link between the recording lever and the glass disk. To secure the natural tones of the piano and organ, the phonograph should be placed at a considerable distance from the instrument. The best records I have got from the organ were obtained when the recording phonograph was about forty feet distant from the instrument. Then the balance of the intensity of the various tones is excellent, and although there is a loss in intensity there is a gain in quality and distinctness.

V. The Mechanism of the Recording Point in the English Model.

9. It was a considerable time before I thoroughly understood the way in which the marker made its impression with each vibration on the wax cylinder. The original tinfoil phonograph was so constructed that when the diaphragm was pressed inwards by the condensation of the air wave, the marker made a corresponding depression on the tinfoil, and when the diminution of pressure came on, corresponding to the rarefaction of the air wave, the marker passed away from the tinfoil. There were thus a series of marks the depth of each of which corresponded to the degree of pressure on the dia-

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phragm. A hasty inspection of the more complicated apparatus in the English model might lead one to suppose that the action in it was of the same nature, but a more careful scrutiny will show that this is not the case. By the model now exhibited you will see that, when pressure is made on the diaphragm, the effect is to cause the cutting edge of the recording gouge to be directed downwards. As the cutting edge of the gouge is directed against the wax cylinder and is opposed to the rotation of the latter, it is evident that this change of the angle of the gouge to a downward direction will cause the gouge to cut a deeper groove into the wax cylinder. The depth of the groove, as determined by the angular movement, is therefore a measure of the pressure on the glass disk. It must be borne in mind that when no pressure is exerted on the glass disk the marker cuts a groove. When there is greater pressure, by the cutting edge being placed at a larger angle with the tangent of the curved surface of the cylinder, a deeper groove will be cut. On the other hand, when the cutting edge is placed at a smaller angle with the tangent of the curved surface of the cylinder a shallower groove will be ploughed on the surface of the wax cylinder. It follows that, if the sound falling on the wax cylinder of the phonograph be very intense, during the increase of pressure the groove will be deep, and during the diminution of pressure the groove will be shallow; and so great may be the difference between the plus pressure of condensation and the minus pressure of rarefaction that during the latter the recording point will only skim the surface of the wax cylinder, without making any groove. This explains an anomaly in several of the photographs taken of portions of the surface of the wax cylinder. For example, a photograph of a portion of a record taken of sound emitted by a full organ shows deep furrows, continued for a considerable distance, corresponding to the long chord-like sounds of the instrument, and these are succeeded by portions in which there is no groove. This is well seen in the fig. 1 (Plate I.). In this case, so great has been the rebound from the state of great pressure that the cutting edge has only slid along the surface of the wax cylinder without cutting a groove. The same is seen when chords are made by bowing several strings of the violin, as in fig. 2 (Plate I.).

10. It is possible that here we have the explanation of one of the imperfections of the phonograph, or, perhaps, rather an illustration of the wrong way of using the instrument. All who have tried the instrument must have observed that the best effects are obtained by tones of moderate intensity. If too weak, the tones given out on reproduction are only imperfectly heard on account of their weak intensity, and by no system of reinforcement or electrical relays can these be made fairly audible. On the other hand, if too strong, there are two risks:—(1st) The intensity of the tone may cause a jarring between the end of the wire in the loop connecting the wire of the lever with the wire from the glass disk, and, as this is communicated to the glass disk, a noise is produced; and (2nd) the intensity of the tone may be so great as to cause, during the rarefaction of the air corresponding to the diminution of pressure, the recording marker to come to the surface of the wax cylinder, or even to leave it altogether. Suppose the marker just skims the surface, it will produce a friction sound which must affect quality, and suppose the

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marker leaves the surface altogether for a fraction of a second, there will be a rebound from the glass disk (owing to the removal of pressure coming from the marker) which is not exactly the same as the diminution of pressure due to the rarefaction of the aerial wave in the immediately preceding vibration. These changes must affect quality of tone.

VI. The Nature of the Marks on the Cylinder

11. Many attempts have been made to obtain tracings of the vibrations of membranes and of glass or metallic disks. In 1856 Leon Scott * invented the well-known phonautograph, which may be regarded as the precursor of the phonograph, and by which vibrations were recorded. Donders,† in 1870, applied the instrument to the investigation of vowel sounds. Next came the logograph of Barlow,‡ by which curves were obtained by the vibrations of a thin membrane of gold-beater's skin. These curves represented the varying pressures of the expelled air taken as a whole, but did not indicate pitch. About 1873 Koenig introduced the method of manometric flames, and flame pictures of vibrations were thus obtained. In 1876 Clarence J. Blake § employed the human membrana tympani as a logograph. In the same year Stein ‖ carried out a method by which he photographed the vibrations of tuning-forks, strings, &c., by attaching to them plates of blackened mica perforated with small holes. A beam of light passing through a hole was allowed to play on a sensitive photographic plate moving with uniform velocity. There was thus recorded a curve representing the combined motions. All of these instruments made it possible to record vibrations, but the sound could not be reproduced from the tracings thus obtained. This was accomplished in 1877 by Edison, by the invention of the phonograph. In 1878 Fleeming Jenkin and Ewing ¶ succeeded in obtaining tracings of the record of vowel sounds on the tinfoil phonograph, and the curves were submitted to harmonic analysis. Shortly afterwards, and in the same year, E. W. Blake ** succeeded in photographing the minute vibrations of a circular ferrotype plate screwed to a telephone mouthpiece by attaching a small mirror to the back of the plate and directing a reflected beam of sunlight on a moving photographic plate. Since that time, the marks on the tinfoil of the first phonograph have been scrutinised by Grützner, Mayer, Graham Bell, Preece, and Lahr.†† The imperfections

* E. L. Scott, *Comptes rendus*, t. 53, p. 108.

† Donders, *De Physiologie der Spraachklanken in het bijzonder van die der nederlandische taal.* Utrecht, 1870.

‡ Barlow, *Trans. of Royal Society*, 1874.

§ Blake, C. J., *Archiv. of Ophthalmology and Otology*, vol. v. 1, 1876.

‖ Stein, S. Th., "Die Photographie der Tone," *Poggendorff's Annalen*, 1876, p. 142.

¶ Fleeming Jenkin and Ewing, "On the Harmonic Analysis of certain Vowel Sounds," *Trans. Roy. Soc. Edin.*, vol. xxviii. p. 145.

** Blake, E. W., "A Method of recording Articulate Vibrations by means of Photography," *Amer. Jl. of Science and Arts*, 3rd ser., vol. xvi. p. 54.

†† Referred to in *The Telephone, the Microphone, and the Phonograph*, by Count du Monce, London, 1884. See also *The Speaking Telephone and Talking Phonograph*, by G. B. Prescott, New York, 1878.

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of the tinfoil phonograph made progress impossible for ten years (from 1878 to 1888), during which time, however, EDISON, GRAHAM BELL and others were engaged in working out the mechanical details of the wax-cylinder phonograph. The subject was then taken up by HERMANN * of Königsberg, and he succeeded in obtaining photographs of the vibrations produced by the vowel sounds, a beam of light reflected from a small mirror attached to the vibrating disk of the phonograph being allowed to fall on a sensitive plate while the phonograph was slowly travelling. The curves thus obtained were very beautiful, and present a striking resemblance to some of KOENIG's flame pictures. In 1891 BOEKE † of Alkmar, in a laborious microscopical research, measured the transverse diameters of the depressions on the wax cylinder at different depths, and from these measurements calculated the depths of the curves. He thus reconstructed the curves on a large scale. The last attempt at recording sound vibrations by photography that has come under my notice is by WILLIAM HALLOCK ‡ of Columbia College, who has succeeded in photographing the flames of an apparatus somewhat similar to the analyser constructed by KOENIG. Manometric capsules were attached to eight resonators corresponding to the eight tones of a harmonic series, and when the flames were lit, by a device of swinging the camera in front of them, a photograph was obtained of the eight bands of flame, as modified by singing the vowels in front of the resonators.

12. I have endeavoured to study the marks on the wax cylinder in three different ways:—(*a*) Taking a cast of the surface of the cylinder; (*b*) taking a microphotograph of a portion of the surface of the cylinder; and (*c*) recording the curves on a slowly moving surface, by a method to be afterwards described.

A. Casts.

13. As regards the first method, taking casts, which was also attempted by HERMANN and BOEKE,§ the results were not satisfactory. The most efficient method followed by me was to paint on the cylinder, with a camel-hair brush, a layer of celloidin dissolved in ether. This soon hardened and the film could then be peeled off. The thin film thus obtained was then inverted on the stage of a microscope, and the marks were seen in relief, as in fig. 3 (Plate I.). This method had the disadvantage of flattening the curves. The depressions are well seen and their differences as regards length are obvious.

* HERMANN, "Ueber das Verhalten der Vocale am neuen Edisonschen Phonographen," *Pflüger's Archiv*, vol. 47, 1890, p. 42; also *Phonophotographische Untersuchungen*, ii. p. 44; also *Phonophotographische Untersuchungen*, iii. p. 347. See also curves of the phonautograph obtained by PIPPING, *Zeitschrift für Biologie*, vol. xxvii. p. 1, 1890.

† BOEKE, "Mikroskopische Phonogrammstudien," *Pflüger's Archiv*, vol. 50, 1891, p. 297.

‡ HALLOCK, "Photographic record of Sound Analysis," *The American Annual of Photography for* 1896, p. 21.

§ Herr BOEKE informs me in a letter that he obtained casts of the surface of the wax cylinder by covering the surface with very thin tinfoil such as is used for covering chocolate.

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B. Photographs.

14. I took numerous photographs, with aid of the microscope and camera, of portions of the surface of the cylinder on which were records of many instruments and of the voice. Specimens of such photographs are shown in figs. 4, 5, 6 (Plate I.). Each figure, from above downwards, represents the $\frac{1}{5}$th of an inch on the surface of the wax magnified fourteen diameters. The grooves seen in each figure are, on the wax cylinder, $\frac{1}{200}$th of an inch apart, and the length of the groove, from above downwards, represents in time the $\frac{1}{64}$th second; that is to say, when each tracing was recorded, the sapphire point of the recorder travelled over the distance represented in magnified proportions in $\frac{1}{64}$th part of a second. By counting the number of indentations or marks, which in a photograph have a curious appearance of being in relief, one can at once determine approximately the pitch of the tone, the vibrations of which make the impression. The tones highest in pitch were obtained from the piccolo and the xylophone, as in figs. 7 and 8 (Plate I.). Here the pitch was about 1920 vibrations per second. In fig. 9 we have a picture of the vibrations produced by the tones of the violin, and it will be seen that they vary in character. Sometimes the marks are a little apart and at other times they blend into each other, the mark widening out as the recording point cut into the wax and then contracting as it receded. It is to be borne in mind that even when the glass disk is not vibrating, the recorder ploughs a groove in the cylinder, and when the glass disk vibrates, each vibration cuts deeper into the groove. The figure of the vibrations of the tones of a flute (fig. 1) shows moniliform markings, indicating that the disk may not, in some instances, return to its position of rest for a short time. In accordance with the description already given of the mechanism of the recording point, it will be seen that, in the figure of the vibrations of the tones of an organ, the intensity of the tone is so great as to cause after each deeply ploughed groove a rebound lifting the recorder on to the surface of the cylinder, or even off the surface altogether. This is the explanation of the smooth spaces between the ends of individual furrows. This action is also well seen in fig. 10, showing the strong vibrations of a tuning-fork. Here one can distinctly observe the slight groove, made by the very tip of the recorder, between the individual vibrations.

15. The accuracy of the phonograph is well illustrated by the four records, figs. 11, 12, 13, and 14. Here we have four records from four of KOENIG'S tuning-forks, the vibration numbers of which are 64, 128, 256, and 512. When the records were taken the cylinder of the phonograph was making as nearly as possible two revolutions per second. Photographs were then obtained of portions of the cylinder representing vertically $\frac{1}{64}$th second. It will be noticed that in fig. 11 there is one mark or vibration in the $\frac{1}{64}$th second; in fig. 12 there are two marks ($64 \times 2 = 128$); in fig. 13 there are four marks ($64 \times 4 = 256$); and in fig. 14 there are eight marks ($64 \times 8 = 512$). As the phonograph travels with remarkable uniformity of speed when it is in good working order, we may rely upon it as an instrument for determining pitch, provided the time

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value of a given length of surface is accurately fixed. The photographic method, however, interesting as it may be, is not of great value, as it does not give the forms of the curves represented by the bottoms of the depressions made by each vibration of the disk of the phonograph.

C. Mechanical Tracing of the Curves.

16. A mechanical representation of the curves presents many difficulties. These were so far overcome by the device of JENKIN and EWING with the tinfoil phonograph. The method followed by these observers, which was entirely mechanical, was to cause the disk of the phonograph to record its movements on a drum moved at the same rate as that of the cylinder. As already mentioned, HERMANN photographed the oscillations of a beam of light reflected from a small mirror connected with the disk of the phonograph, the whole apparatus moving slowly. My method consisted in the adaptation of a light lever to a marker connected with the phonograph itself, and so arranged that it (the point of the marker) would travel over all the ups and downs of the phonographic curve on the wax cylinder at an extremely slow rate. The obvious objections to any method of directly recording the ups and downs of the lever is that the inertia of the lever might cause extraneous vibrations, while at the same time the smaller marks on the wax cylinder might be missed. These objections, however, may be removed by (*a*) reducing friction to a minimum, and (*b*) moving the phonograph cylinder so slowly as to make the movement almost invisible to the naked eye. In this way inertia ceases to be a trouble.

17. After various attempts with simple appliances, the apparatus shown in Plate I. was fitted up, and by means of it the taking of curves became comparatively easy.* In fig. 16, Plate I., a diagram of the arrangement is shown as simply as possible. The first apparatus used was not specially made for the experiment, but consisted of fittings in my own laboratory. The motive power for driving the apparatus is a small water motor acting on a wooden wheel. From this wheel two trains of wheels or pulleys pass, the one set, namely, 2, 3, 4, and 5′, being gradually geared down so as to drive the cylinder of the phonograph, 6,† at an extremely slow rate, and the other set, namely, 2′, 3′, and 4′, to drive at a slow rate the recording drum *a*. By this arrangement the rate of rotation of the cylinder of the phonograph is about once in five or six *minutes*, instead of being once in one-half second, the usual speed in recording. The drum, *a*, also moves very slowly, but a little faster (not twice as fast) than the cylinder of the phonograph, the object being to open out the curves somewhat in a linear direction. By attaching an electric arrangement to the axle of the mandril carrying the phonograph cylinder, 6, the time of each revolution of the phonograph cylinder was registered on

* This apparatus has already been described in a short paper written for the *Journal of Anatomy and Physiology*, vol. xxix.

† The cord passes over (*b*), the wheel seen on the left end of the phonograph spindle in the figure showing the instrument. The wheel will be recognised by a tape band passing vertically over it.

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the drum *a*. Thus the amount of surface on *a*, representing one-half second on cylinder 6, could always be measured. As a rule, it was found that 3 inches of the drum *a* represented one-tenth of a second. Occasionally it was one-seventh of a second, but it could be so timed as to represent one-tenth.

18. In the next place, a light lever of hard wood, braced like the mast of a ship, was fixed firmly into a socket bored into the lead weight, *i k*, seen in fig. 16, Plate I. This square leaden weight is hinged to the frame carrying the recording and reproducing part of the phonograph by the hinge *h*, a slot is cut in the under surface of the lead weight, as seen in fig. 17, Plate I., and the marker, *a n* (also *m n* in fig. 16), moves on an axle delicately pivoted to the sides of the slot. In fig. 16, at the end of *m*, is seen the wire, *w*, passing up to the glass disk, *d*, of the phonograph. It will be seen that the point of *m n* touches the surface of the wax cylinder, *o*, *o*, *o*, *o*. The lead weight accurately follows the movements of *m n*, the marker, and the lever *s* records these movements, as seen in fig. 15, on the drum *a*. The slowness of movement does away with any movement of *s*, except that which is communicated to it from the lead weight and from the marker, *m n*, and to prevent all extraneous vibrations the phonograph was seated on a solid stone pillar erected for a galvanometer. The point of the lever *s* is the point of a very fine hard needle, and *a* was covered with very smooth paper, carefully smoked. As the marker of the phonograph is always travelling to the one side so as to describe a spiral groove, that is, towards the observer, when one looks at fig. 15, it is evident that the point of the lever *x* would soon leave the surface of *a*. To get rid of this difficulty, the drum *a* is mounted on a kind of movable table moving in slots, and controlled by the fine screw *c* (one thousand to the inch thread). And thus it was easy, by a turn of *c*, to keep the point of *x* in contact with the surface of *a*. Finally, the movable table rested on a plate of metal moving vertically in slots by turning the fine screw *b*, and thus, after a tracing had been taken once round *a*, it was easy to move *a* a little up or down without disturbing the lever *x* (fig. 15). Thus I had three motions of *a*, vertical, rotatory, and horizontal, and there were two movements of the cylinder of the phonograph *b*, rotatory and horizontal. Everything was as steady as it could be made, and the apparatus worked almost automatically, the point *n* (fig. 16) slowly *crawling* over the surface of the wax cylinder *o*, about 600 times slower than when the record was made on the wax cylinder.

19. After tracings had been taken, the paper on *a* was varnished and it was cut into longitudinal strips, each 3 inches in length, and the strips were mounted between two ordinary microscope slides (English make) in the way that slides are prepared for the lantern. Such slips can then be examined by the microscope and the curves may be drawn by ABBE'S camera lucida, or any portion of the tracing may be photographed by a convenient microphotographic apparatus. Thus the size of the curves may be much increased, and so made available for purposes of harmonic analysis.

20. This apparatus, which was large and clumsy, was discarded for the small and more convenient apparatus, consisting of a train of four sets of wheels and

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pinions, so geared as to obtain a gradually diminishing rate of motion. Pinions on the axles of the wheels serve for cords by which the phonograph and also the recording drum may be driven at various rates of speed. This apparatus may be so worked as to cause the cylinder of the phonograph to perform one revolution in one hour instead of in half a second. The lever gave an amplification of seventy times, and as many of the marks obtained on the recording cylinder were only about $\frac{1}{30}$th inch in vertical height, it follows that the depths of many of the depressions on the wax cylinder were not more than $\frac{1}{2000}$th of an inch.

21. It has already been pointed out that with the view of opening out the curves the recording drum was caused to move faster than the phonograph cylinder. Unless this had been done, the curves of the individual vibrations would have crowded so close together as to make it impossible to view them distinctly. This will be evident if we look at one of the photographs (fig. 4, Plate I.), representing in vertical length the one-fifth of an inch. In this distance we may have from ten to twenty depressions, and if these had been recorded on the same longitudinal extent of surface on the recording cylinder, we would have had ten to twenty little waves in the one-fifth of an inch. But, if an analysis of the curves has to be undertaken, it is not necessary to have the cylinder of the phonograph and the recording cylinder travelling at the same rate. Even although the latter moves faster than the former, this is of no consequence as the coefficients of the harmonic series are altered all in the same ratio. It does not matter what may be the *form* of the curve representing an individual vibration. This might be either an exact facsimile of the depression on the wax or it may be drawn out by causing the recording drum to move faster. Whatever be its form, if the curve is built up of a series of curves that are members of a harmonic series, these constituents can always be determined.

22. Specimens of the curves thus obtained are shown in figs. 8 to 18, Plate II. It will be seen that they vary much in character.

VII. A Method of Recording Variations of Intensity of the Sounds of the Phonograph.

23. Suppose a series of sound waves of gradually increasing intensity to fall on the disk of the phonograph, the pressure on the disk will gradually increase and the normal groove will be cut deeper. In this process each vibration will be a little deeper than the one immediately before it, but the difference in depth will be very small. If the increase of pressure of the note or chord lasted more than half a second, the extent of surface covered by the recording point during that time would be nearly 7 inches, and there might be from 500 to 1000 depressions in that distance. Suppose, now, that we recorded all these little depressions by the slow moving lever method, it will be evident that the gradually increasing differences in height of the little curves would scarcely be appreciable.

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The slow method of recording vibrations, therefore, whilst it is the method by which data can be obtained that have to do with pitch and quality, will fail in giving us a record of variations in intensity. This aspect of the matter came under my notice at an early period of the investigation. So far as I am aware, no one has attacked this side of the problem. Nothing is more striking in listening to the phonograph when it is reproducing either human speech or musical sounds than the way in which it catches every inflection of the voice or the slightest emphasis, diminuendo, and crescendo of the sound. This must be due to variations of pressure. How may these variations be recorded?

24. The most evident method is to attempt to record mechanically the variations in an electro-magnet produced by pressures on a variable resistance apparatus in the same circuit. My first attempt was to place GRAHAM'S transmitter over the glass disk of the phonograph and to place in the same circuit an electro-magnetic marker such as is used for physiological purposes. This gave poor results, but still they were encouraging. On placing a BREGUET'S chronograph in circuit the results were much better, and it was evident that there was a movement of the vibrator of the chronograph for each note or chord emitted by the phonograph. I then heard of an ingenious apparatus devised by HEURTLEY of Breslau, by which he has succeeded in recording by electrical and mechanical arrangements the sounds of the heart. His apparatus consists essentially of a large stethoscope on which a peculiar resonator is fixed. The resonator carries a small wooden tuning-fork, between the prongs of which is fixed a simple microphonic contact of two carbon buttons. This is one half of the apparatus. The other half consists of an electro-magnet, over the poles of which is fixed, face downwards, a shallow tambour, of the Marey pattern, having on its under surface a broad ferrotype plate. This tambour is then connected with an extremely delicate recording tambour. When I heard of this apparatus I at once saw that the second half of it was exactly what I wanted for the phonograph work, and, by the kindness of Professor HEURTLEY, the apparatus was made for me in Tubingen without delay. When placed in the circuit along with the carbon transmitter the pen of the recording tambour moves at right angles to the line of revolution of the cylinder with each tone and chord played by the phonograph. When the ear perceives tones of considerable intensity the lever point is seen moving through a greater distance than when the tones are weaker; consequently we have a graphic record of the variations in intensity. If the recording cylinder is timed to travel at the same rate as the cylinder of the phonograph, then the curves on the former exactly correspond to the ensemble of the minute marks on the latter corresponding to a particular variation in intensity. When the recording cylinder is caused to travel as fast as the phonograph cylinder, the variation in the heights of the curves recorded on the revolving cylinder is not so apparent as when the recording cylinder travels more slowly. It is easy, however, to time the rate of revolution of both cylinders by a chronograph. Thus I have found that when the recording cylinder is travelling at such a rate that $\frac{1}{4}$ inch of surface corresponds to one-fourth of a second, an

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easily read tracing is obtained. In such a distance we may have one little wave representing the pressure of a chord lasting for one-fourth of a second, or we may have from two to as many as fifteen little waves, often varying much in general character. Suppose we find as many as fifteen; then each must have lasted not more than $\frac{1}{60}$th of a second. Even then the ear is able to follow the individual notes, when the phonograph is listened to simultaneously. This may be readily done either by listening directly to the phonograph or by connecting a telephone with the secondary of an induction coil, while the current in which the variable resistance apparatus is interposed passed through the primary. If, then, we hold the telephone to the ear while we look at the little pen writing on the recording drum, it is easy to see that the sensations are simultaneous. Now if a note of a pitch say of 300 vibrations per second lasts only $\frac{1}{60}$th of a second, it is evident that only five vibrations must have occurred in that time. This shows that we can appreciate a tone and decide as to its pitch if only five vibrations fall on the ear. This conclusion coincides with the opinion I arrived at during last summer from a careful inspection of the photographs and of the mechanically recorded curves. Of course I assume that the music is being played by the phonograph in its proper *tempo*. If the phonograph is made to travel faster, possibly it might be found that pitch might be appreciated for even shorter periods. Examination of the curves shows that as a rule no "chord" lasts longer than half a second. This method of recording seems well suited to the study of the time relations if a series of complex sounds pour in upon the ear.

25. If one doubts whether the movements of the recording lever coincide with the tones of the phonograph, three ways are open by which the statement may be put to the test:—(1) Listen attentively with the telephone and at the same time watch the recording point. The sensations of hearing and of vision for any particular note are simultaneous. (2) Remove the elastic tube from the recording tambour and place it in the ear and the music will be heard. (3) Lead the elastic tube from the electric tambour to a recording phonograph, and a feeble record will be obtained of the music showing that all the vibrations are present. In the two last experiments, as might be expected, quality suffers, but the rhythm, the *tempo*, and the general character of the tune are reproduced. Examples of such tracings are shown in fig. 5 (Plate II.).

VIII. The Effect of Reversing the Action of the Phonograph.

26. Long ago, with the tinfoil phonograph it was noticed that, with speech, if the movement of the cylinder was reversed, there was a reversal of the sounds. I recollect that in 1878 Lord Kelvin and I often repeated this experiment. Recently I have re-examined this matter with the aid of a phonograph in which a reversed movement has been obtained by a change in the position of the brushes of the electric motor that drives the machine. It will be observed that this is the most evident way of obtaining a reversed action. It will not be obtained by simply reversing the wax cylinder on the india-rubber mandril of the machine, because, in the first place, the wax cylinder is of

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unequal thickness at each end to suit the tapering form of the mandril and it cannot be put on in a reverse position, and, in the second, when the wax cylinder is in its normal position a spiral groove is cut from left to right, and if it were reversed it would pass from right to left, while the feeding screw of the machine which secures the motion from left to right would still be moving in that direction. Having secured the reversal, it is easy to show that words pronounced backwards to a phonograph moving in the direction of left to right and then reproduced from a phonograph moving in the direction of right to left are uttered normally and with considerable distinctness.

Thus the sounds—

Fargonof	become	Fonograf
Nōfelet	,,	Telephōn
Fārgelet	,,	Telegrāf
Poksèlet	,,	Teleskop
Arrubnidē	,,	Edinburrā
Wogsalg	,,	Glasgow
Elponitnatsnoc	,,	Constantinople
Eetisrevinū	,,	Universitēe.

The difficulty, of course, is in emphasising the proper syllables. In pronouncing a word backwards not only may the emphasis be placed on the wrong syllable but suppose that in an emphasis one begins the sound crescendo and ends it off diminuendo, when the reversal takes place we have the opposite, often giving a ludicrous effect. This is well observed on listening to reversed music. Not only are all one's notions of the relations of tones thrown into confusion, but as with many instruments the tone is sharply and distinctly taken and then is allowed to weaken in intensity, the effect is produced of tones beginning diminuendo and ending loudly and abruptly. Still it is interesting to find that such tones as those of the vowel sounds, flute tones, cornet tones, and forks, if spoken, sung, or played smoothly, and, as far as possible, with a uniform intensity, come out with equal distinctness and, so far as I can observe, with very little if any alteration in quality, whether they are taken normally or reversed. No experiment could illustrate more strikingly the dependence of quality of tone on the resultant wave forms imprinted on the cylinder of the phonograph. At the same time, I am not convinced that with the means at our disposal we have yet obtained an absolutely accurate reversal. This criticism was first suggested to me by Professor TAIT some time ago, and a careful scrutiny of the marks shows that he is right. The depression made on the wax cylinder for a given vibration does not seem to be deepest in the centre, but the maximum dip is nearer one end while there is a tail or "trail out" as the pressure is removed. The direction of the trail out is opposed to that in which the cylinder is travelling, and as the cylinder travels with great velocity, the trail out is quite appreciable. Were the phonograph a perfect machine, the maximum depth of the curves made by compound tones or by harmonies would be in the centre, and the form of the curve, corresponding to the periodic variations of air pressure, would depend on the sum of its

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components varying separately according to the simple harmonic law. If we take as an example a harmony, the variation of the air pressure of a harmony is the sum of the variations of simple tones, one having a period equal to the period of the harmony, a second $\frac{1}{2}$, a third $\frac{1}{3}$, and so on.* It may be that this defect in the form of the curve in the phonograph is the explanation of the fact that it does not, in many cases, reproduce absolutely the quality of tone. Further, when reversal takes place, it will be evident that the reproducing point will run down the slope that corresponds to diminution of pressure before it runs up the slope corresponding to increase of pressure, and that it will take longer time in going down than in going up the slope. This, again, must affect quality.

IX. Remarks on Discrimination of Auditory Sensations.

27. This study of the phonograph has led to the consideration of some of the fundamental questions of physiological acoustics, but I shall briefly refer to only one of these at present. It is clear that by the aid of the phonograph we can record vibrations of higher number than by any other method at our disposal, and that we can see and study the time relations of these vibrations. This is a direction in which research should proceed. So far as I am concerned, I must leave it to others. By this method, for example, direct observations might be made on the limits of the sensible discrimination of tones. It is well known that the lower limit of tone perception is near 16, and the upper limit near 50,000 vibrations per second, and it is also well known that sensible discrimination diminishes towards each end of the range, and more especially towards the upper limits. By noticing the smallest changes in different determinations, and by the method of computing right and wrong cases in observations of the comparison of differences, experimental psychologists have shown that, between 64 and 1024 vibrations per second, the least noticeable difference is so small as 0·2 vibration. It is said to be 0·4 at 32 and 2048 vibrations per second. It is certainly a remarkable testimony to the delicacy of the ear that, between 32 and 2048 vibrations per second, we can observe a difference of less than 1. Even the most skilled ear cannot observe large differences between 12,288 and 16,384, and Külpe states that only about 23 tones can be distinguished beyond 4096. From these data, we can compute, as has been done by Külpe,† the number of tones that the ear can distinguish. Thus, in the range of human audibility, we can hear about 97 tones between 16 and 64 vibrations per second, 4800 between 65 and 1024, 6144 between 1025 and 4096, and 23 on the high side of 4097. This gives a total of 11,064 tones heard by the human ear. It is interesting, in this connection, to notice the range as regards pitch of the chief instru-

* Lord Kelvin, "On Beats of Imperfect Harmonies," *Proc. Royal Society of Edinburgh*, 1st April 1878; also in *Popular Lectures and Addresses*, vol. ii. p. 395.

† Oswald Külpe, *Outlines of Psychology, based upon the results of experimental investigation*, p. 106, trans. by E. B. Titchener, 1895.

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ments, as shown in the diagram on Plate III., showing the pitch of the highest and of the lowest tones. The highest tone is that of the piccolo stop of the organ = 4096 vibrations per second, while the lowest is the deepest tone of the contra-bassoon, the vibration number of which is 27.

28. Now the interesting question arises, which was discussed by HELMHOLTZ with the data then at his disposal, of how the ear appreciates this vast number of tones, and how it is that we have the power of analysing compound periodic vibrations into their simplest constituents. Further, it is well known that the tones we thus sift from each other by analysis correspond to the simple vibrations that are shown by mathematical physics to exist in the compound movement. Now we know that such an analysis can be made by a suitably arranged acoustical apparatus, and it seems to follow that probably in the cochlea we might find structures presumably capable of doing such work. Without discussing possible theories of the cochlea, we may assume that it might work in one of two ways. Either small vibratile bodies would be introduced between the pressures sent into the organ and the filaments of the auditory nerve, or that individual nerve fibres were directly excitable by waves of a definite wave length, and that they thus had a selective action. It will be observed, however, that in either supposition the individual nerve fibres would convey impulses corresponding to the number of vibrations of the tone selected, as ably argued by Professor RUTHERFORD,* and it does not follow that the number of nervous impulses sent along the filaments of the auditory nerve would be independent of the number of stimuli given to the specially tuned vibratile bodies that are supposed, *ex hypothesi*, to be at their commencements.† To aid, then, in discussing this question, it is of interest to ascertain whether, as a matter of fact, and without considering their mode of action, there are a sufficient number of bodies in the cochlea presumably capable of acting as intermediate structures. I have collected data on this point both from observations and measurements made in my own laboratory, and from the exhaustive account of the cochlea given by RETZIUS.

1. *Audible sounds* = 11,064
   Less those below 64 and above 4096 = 110
   Leaving . . 10,954
   Say . . 11,000

2. *Distribution of audible sounds in six octaves used in music.*
   $\frac{11,000}{6} = 1833$ each octave; $\frac{1833}{12} = 153$ each semitone.
   Musician's ear can detect difference of pitch in tuning a violin of $\frac{1}{64}$ semitone.
   $\frac{153}{64} = 2{\cdot}4$ for each $\frac{1}{64}$ semitone.

1. *Nerve fibres in human auditory nerve*, 14,000.
   Range of audibility, 11 octaves.
   $\frac{14,000}{11} = 1273$ fibres for each octave.

* RUTHERFORD, *Address to British Association for Advancement of Science*, 1887.

† HELMHOLTZ, *Sensations of Tone*, trans. by A. I. ELLIS, p. 221.

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Assuming that the number of auditory filaments is the same for each of the eleven octaves, which is unlikely, the probability being that there will be fewer for the range below 64 and above 4096.

$1273 \times 6 = 7638$ for the six octaves under consideration.

$\frac{1273}{12} = 106$ fibres for each semitone.

$\frac{106}{64} =$ *Less than 2 fibres for each $\frac{1}{64}$ of a semitone.*

2. *Fibres in Membrana basilaris*, 24,000.

$\frac{24,000}{11} = 2182$ fibres for each octave.

$2182 \times 6 = 13,092$ for the six octaves under consideration.

$\frac{2182}{12} = 180$ fibres for each semitone.

$\frac{180}{64} =$ *Less than 3 fibres for each $\frac{1}{64}$ of a semitone.*

3. *Hair Cells: Inner*, 3487; *Outer*, 11,750 = 15,237.

$\frac{15,237}{11} = 1385$ cells for each octave.

$1385 \times 6 = 8310$ hair cells for the six octaves under consideration.

$\frac{1385}{12} = 115$ cells for each semitone.

$\frac{115}{64} =$ *Less than 2 for each $\frac{1}{64}$ semitone.*

4. *Corti's Rods: Inner*, 5590; *Outer*, 3848 = 9438.

$\frac{9438}{11} = 858$ rods for each octave.

$858 \times 6 = 5148$ rods for six octaves under consideration.

$\frac{858}{12} = 71$ for each semitone.

$\frac{71}{61} =$ *at least one rod for each $\frac{1}{64}$ semitone.*

As there is a high probability that there are fewer nerve fibres for the five octaves left out of consideration, and consequently more in the six octaves used in music, it follows that we have in the cochlea a sufficient number of possibly vibratile masses to satisfy the demands of theory.

29. The view that the cochlea is a differential apparatus is also supported by the fact that careful measurements made of the same parts of the organ in man, the cat, and rabbit show considerable variations, variations that correspond also with all we know of the auditory powers of these examples. These measurements also help to give one a conception of the relative dimensions of different parts of the organ, and, to make this more obvious, I have given a column in which the measurement has been multiplied by one hundred. These measurements have been collected chiefly from the works of RETZIUS :—

a. *Lengths of Cochlear portion of Internal Ear in Man, Cat, and Rabbit.*

	Membrana basilaris and papilla basilaris.			*Ductus cochlearis.*		
	mm.	inch.	100 times.	mm.	inch.	100 times.
Man, .	35·5	1·42	142	36	1·44	144
Cat, . .	23·5	0·94	94	25	1·00	100
Rabbit, .	15·0	0·60	60	16	·64	64

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b. *Radial Breadth of Vestibular Wall.*

	mm.	inch.	1000 times.
Base,	·81	·0324	32·4
Middle,	·88	·0325	32·5
Apex,	·85	·0340	34·0

Vestibular wall increases from base to apex.

c. *Radial Breadth of Tympanic Wall.*

	mm.	inch.	1000 times.
Base,	·45	·018	18
Middle,	·77	·030	30
Apex,	·80	·032	32

Tympanic wall increases from base to apex.

d. *Height of Outer Wall.*

	mm.	inch.	1000 times.
Base,	·585	·023	23
Middle,	·500	·020	20
Apex,	·350 to ·4	·030	30

Outer wall diminishes towards the centre and then increases.

e. *Height of the Tunnel.*

	mm.	inch.	1000 times.
Base,	·028	·00112	1·12
Middle,	·045	·00180	1·18
Apex,	·049	·00196	1·96

Tunnel becomes higher as we pass from base to apex.

f. *Radial Breadth of the Membrana Basilaris (from Habenula Perforata to the Spiral Ligament).*

	mm.	inch.	1000 times.
Base,	·21	·0084	8·4
Middle,	·34	·0120	12·0
Apex,	·36	·0144	14·0

The basilar membrane increases towards the apex.

g. *Radial Breadth of Limbus Spiralis.*

	mm.	inch.	1000 times.
Base,	·243	·0972	97·2
Middle,	·230	·0920	92·0
Apex,	·225	·0900	90·0

The spiral bony shelf diminishes towards the apex as the basilar membrane increases.

Thus, magnified 1000 *times. In inches.*

	Limbus.	Basilar Membrane.	Total.
Base,	97·2	8·4	105·6
Middle,	92·0	12·0	104·0
Apex,	90·0	14·0	104·0

h. *Pillars.*

Length of Inner Pillars.

	mm.	inch.	1000 times.
Base,	·048	·00172	1·72
Middle,	·068	·00272	2·72
Apex,	·070	·00280	2·80

Length of Outer Pillars.

Base,	·062	·00248	2·48
Middle,	·100	·00400	4·00
Apex,	·103	·00412	4·12

As we pass from base to apex both pillars lengthen.

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i. *Hair Cells.*

Length of Inner Hair Cells.

	mm.	inch.	1000 times.
Base,	·018	·00072	·72
Middle,	·024	·00096	·96
Apex,	·024	·00096	·96

Length of Outer Hair Cells.

Base,	·03	·0012	1·2
Middle,	·04	·0016	1·6
Apex,	·04	·0016	1·6

As we pass from base to apex, both inner and outer hair cells increase in size.

k. *Length of the Hairs of the Hair Cells.*

Hairs of Outer Cells.

	mm.	inch.	1000 times.
Base,	·0045	·00018	·18
Middle,	·0060	·00024	·24
Apex,	·0080	·00022	·22

Hairs of Inner Cells.

Base,	·0045	·00018	·18
Middle,	·0055	·00022	·22
Apex,	·0055	·00022	·22

(1) *In both, the hairs increase in length from base to apex.*
(2) *The increase is greater in the hairs of the outer cells.*

l. *Radial Breadth of Membrana Tectoria, covering Corti's Organ.*

	mm.	inch.	1000 times.
Base,	·285	·0114	11·40
Middle,	·340	·0136	13·60
Apex,	·345	·0138	13·80

It increases from base to apex, like the basilar membrane, but it is not far off one-third of the breadth of this membrane.

m. *Comparison of Man, Cat, and Rabbit.*

	Man.	Cat.	Rabbit.
Ear teeth,	2,490	2,430	1,550
Holes in Habenula for nerves,	3,985	2,780	1,650
Inner rods,	5,590	4,700	2,800
Outer rods,	3,848	3,300	1,940
Inner hair cells (one row),	3,487	2,600	1,600
Outer hair cells (several rows),	11,750	9,900	6,100
Fibres in basilar membrane,	23,750	15,700	10,500

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DESCRIPTION OF PLATES.

Plate I.

[In Figs. 1–14 (except Figs. 3 and 6), the vertical length of the tracing represents $\frac{1}{64}$th sec., and the cylinder rotates from the bottom to the top of each tracing.]

Fig. 1. Full organ, taken 40 feet from instrument. Portion of Mendelssohn's Wedding March.
Fig. 2. Chords on violin.
Fig. 3. Portion of a celloidin cast of cylinder on which was recorded a march by a military band.
Fig. 4. Flute.
Fig. 5. Tenor voice.
Fig. 6. Violin, quick playing.
Fig. 7. Piccolo.
Fig. 8. Xylophone.
Fig. 9. Tuning fork.
Fig. 10. Tuning fork, 64 vibs. p. sec. (one long mark).
Fig. 11. Tuning fork, 128 vibs. p. sec. (two marks).
Fig. 12. Tuning fork, 256 vibs. p. sec. (four marks).
Fig. 13. Tuning fork, 512 vibs. p. sec. (eight marks).
Fig. 14. Tenor voice (second record).
Fig. 15. Diagram of recording apparatus described in text.
Fig. 16. Shows on a larger scale than in fig. 15 the recording arrangement.
Fig. 17. Under surface of leaden weight to which recording lever was attached.
Fig. 18. Facsimile of curves obtained from record of old English coach horn.
Fig. 19. Facsimile of curves obtained from record of a military band.

Note on Figs. 18 *and* 19.—These curves (3 inches of the tracing representing $\frac{1}{10}$th sec.; that is to say, the curves in 3 inches were recorded on the cylinder of the phonograph in $\frac{1}{10}$th sec.) should be examined with a magnifying glass. In the two upper lines of 18, note the uniform character of the curves. In the third line the character of the curve alters. These are tracings of the marks produced by the long-drawn-out, clear, piercing tones of the horn. In 18, the upper and lower lines show great variety of curve form. The middle line shows no curves, and probably represents an interval of silence.

Plate II.

[*Note.*—Tracings in Figs. 1, 2, 3, and 4 to be read from right to left.]

Fig. 1. Facsimile of the vibrations in the words "Tit for Tat."
Fig. 2. Facsimile of the vibrations in the word "University." See description of next fig.
Fig. 3. Facsimile of the vibrations in the words "Glasgow" and "University." The word "Glasgow" is in the upper line, and it will be observed that the marker was raised a little to the left of the beginning. The word "University" is in the lower line, "Univer" to the right, then a short pause, and then the end of the word, "sity."
Fig. 4. Facsimile of the vibrations in the word "Glasgow."
Fig. 5. Tracings taken with the apparatus described in paragraph 24 of the paper. They are to be read from left to right. They represent the variations in intensity of the notes and chords of a military band. Each wave represents a note or chord, and the height of the wave indicates the intensity. A length of one-quarter of an inch represents the impulses communicated from the phonograph to the microphone in one quarter of a second, that is, in the time occupied by half a revolution of the phonograph cylinder when the sound is reproduced. A careful examination will show that portions of the music are repeated. See the uniform waves in lines 3 and 6. These represent blows on an anvil, as produced in the "anvil chorus," of which this is a reproduction.

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Note.—Figs. 6–18 are from microphotographs of curves similar (though varying in amplitude) to those in figs. 18 and 19 in Plate I., and in Figs. 1, 2, 3, and 4 of this plate. They are magnified from 5–10 diameters. Only short portions of the curves are given to illustrate the great variety in form.

Fig. 6. Tuning fork, 32 vibs. p. sec.
Fig. 7. Cornet.
Fig. 8. Old English coach horn.
Fig. 9. Bassoon.
Fig. 10. Bassoon.
Fig. 11. Cornet.
Fig. 12. Military band.
Fig. 13. Military band.
Fig. 14. Flute.
Fig. 15. Piccolo.
Fig. 16. Piccolo.
Fig. 17. Piccolo.
Fig. 18. Coach horn.

Note.—The curves shown in Figs. 19, 20, 21, and 22 were taken with a syphon-recorder, to be described in a subsequent communication. It will be seen that they are a great advance on the curves obtained by the older methods. One inch of these tracings represent $\frac{1}{100}$ of a second. By counting the number of waves in this distance, the pitch can be determined.

Fig. 19. Cornet.
Fig. 20. Piccolo.
Fig. 21. Curves of a noise. From the record of the noises in a boiler-maker's shop. Note the irregularities of the curves.
Fig. 22. Song by a tenor voice. Note the gradation from one pitch to another. At the right and left ends of the tracing, the voice was sounding a note a little above the middle C, and in the middle the pitch was about an octave lower.

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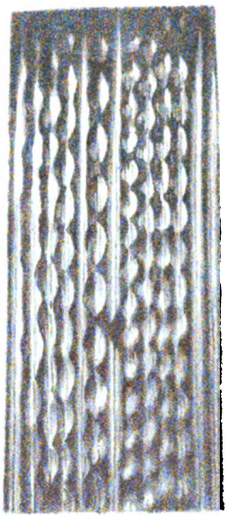

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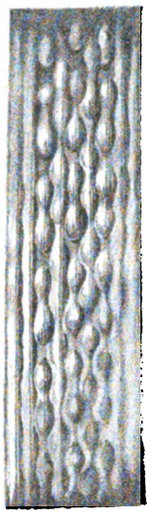

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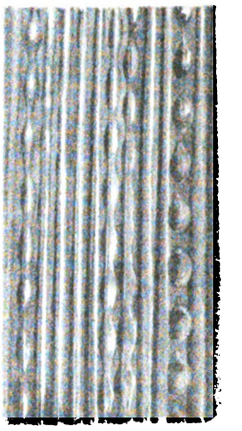

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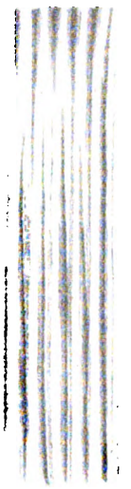

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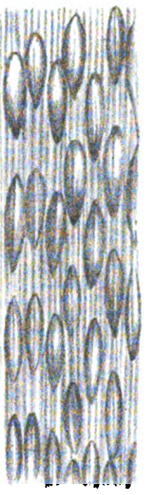

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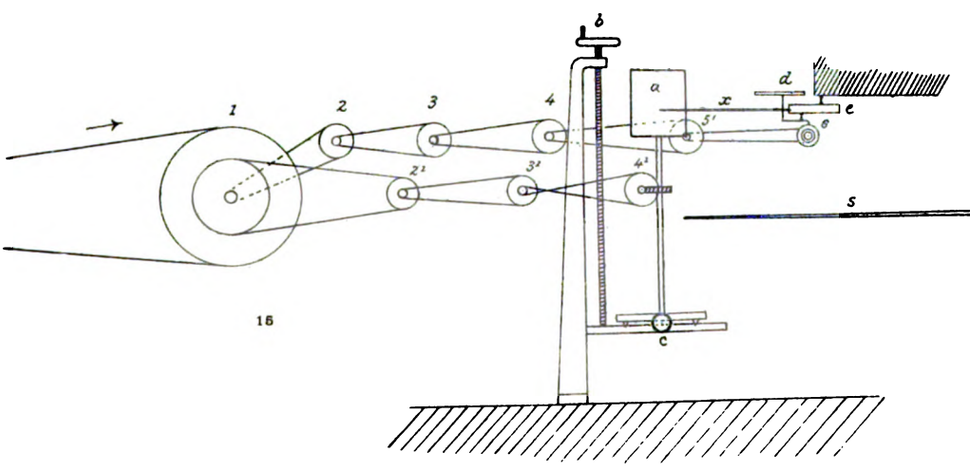

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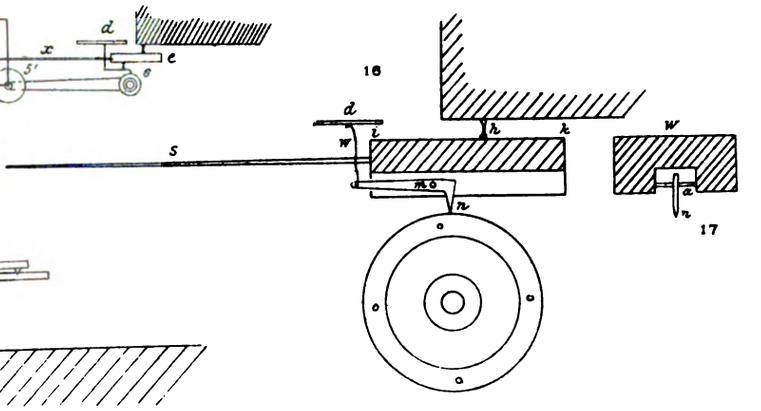

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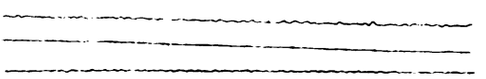

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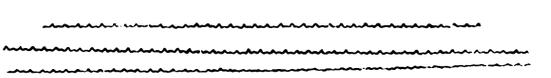

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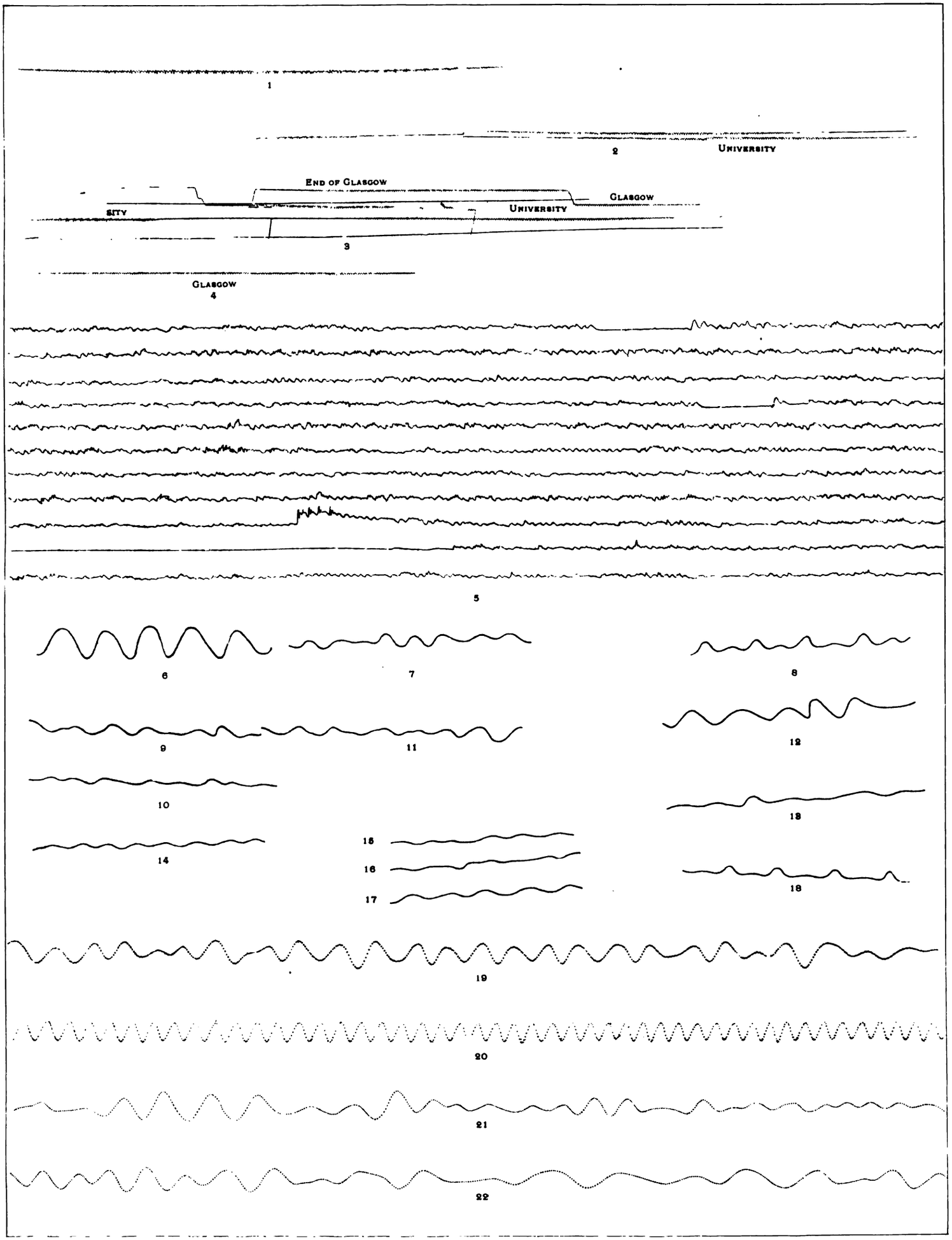

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XXIII.—*On the Genus* Anaspides *and its Affinities with certain Fossil Crustacea.* By W. T. CALMAN, B.Sc., University College, Dundee. Communicated by Professor D'ARCY W. THOMPSON. (With Two Plates.)

(Read 18th May 1896.)

The genus *Anaspides* was founded in 1894* by Mr G. M. THOMSON of Dunedin, New Zealand, for the reception of a very remarkable Schizopod Crustacean which he had discovered in a fresh-water pool at an altitude of 4000 feet on Mount Wellington in Tasmania. The very striking peculiarities of the animal, the absence of a carapace, the presence of plate-like gills attached to the bases of the thoracic legs, and the possession of an auditory organ in the peduncle of the antennules, led its discoverer to regard it as the type of a new family of Schizopods, the *Anaspidæ*, while suggesting that it might be entitled to "even higher specific rank." I have had an opportunity of examining three specimens of *Anaspides* presented to the Museum of University College, Dundee, by Dr CHAS. CHILTON of New Zealand, and from the dissection of one of these I have been able to supplement, in some important points, Mr THOMSON's account of the external anatomy of the animal. I wish also to call attention to the remarkable resemblance, indicative I believe of close affinity, which *Anaspides* bears to certain Palæozoic Crustacea belonging to a group hitherto supposed to be unrepresented among living forms. The present paper has been prepared under the direction of Prof. D'ARCY W. THOMPSON.

MORPHOLOGY OF ANASPIDES.

Body.—When Mr THOMSON describes *Anaspides* as possessing eight free thoracic segments, he makes no mention of the extreme interest and importance which would attach to such a state of things. In no living Malacostracan, save *Nebalia* and certain Stomatopods, do we find more than seven segments of the thorax completely defined, the segment corresponding to the first pair of maxillipeds being fused with the head even where, as in the *Edriophthalmata*, there is no other trace of a carapace. Mr THOMSON's comparison of the body of *Anaspides* with that of an Amphipod would not therefore be strictly tenable, since, on his own showing, there is a difference in the number of segments. We believe, however, that this anomaly does not exist, and that *Anaspides* in this respect forms no exception to the rule among *Malacostraca*. Careful examination shows that the line which, in Mr THOMSON's figure, marks off the "first

* *Trans. Linn. Soc. Zool.* (2), vi. 3. A preliminary account, without figures, was published in *Proc. Roy. Soc. Tasmania*, 1892.

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thoracic" segment from the head, does not correspond to a movable articulation like those which separate the other segments of the body, but to a superficial groove in the integument (Plate I. fig. 2, *c*). That this groove does not represent the line of junction of the first thoracic segment with the head is, we think, shown by the fact that, instead of passing directly downwards on each side, parallel to the hinder margin of the segment, it runs obliquely forwards and ends just behind the root of the mandible, so that between it and the posterior border of the segment is included the region of the two pairs of maxillæ in addition to that of the maxillipeds (Plate I. fig. 1). Mr THOMSON's "first thoracic segment" must represent, therefore, the fused segments corresponding to these three pairs of appendages. Further, the position of this groove is precisely that of the "cervical sulcus," which, in the *Mysidæ*, crosses the carapace immediately above the mandibles, and with this sulcus it must, we think, be identified. And, as there is no reason to suppose that the cervical sulcus of the *Mysidæ* is other than the representative of the cervical groove of the Decapods, we are led to the presumption that the segments bearing the two pairs of maxillæ are morphologically posterior to this groove in the Decapods also, a presumption against which no anatomical evidence appears to militate. And if all this be true, we should then perceive the existence alike in *Anaspides*, in the *Mysidæ*, and in the Decapods, of a primary sulcus delimiting an anterior region or head to which three pairs of appendages, antennules, antennæ, and mandibles belong, the region in fact of the three paired appendages of the Nauplius, the "primary head region" of the Crustacea according to CLAUS.*

From this cervical groove in *Anaspides* there runs back on each side a horizontal line (fig. 2, *a*) which marks off inferiorly a quadrilateral area. Just behind the cervical groove a faint impressed line can be traced (fig. 2, *b*), for the most part nearly parallel with the main groove, but bending backwards for a short distance in the dorsal region, and terminating on each side near the junction of the horizontal line with the cervical groove.

While unable to suggest any definite interpretation of these various markings, we may note that Mr THOMSON's comparison of the horizontal line with that which marks off the "epimeron" on the thoracic segments of various *Isopoda* seems to be quite untenable. The "epimeron" in that group represents the coxal joint of the leg, and it is never marked off on the first thoracic segment, which in the *Isopoda* is indistinguishably fused with the head.

On the dorsal surface of the cephalic region, a short distance in front of the cervical groove, a small ill-defined area is more darkly pigmented than the surrounding parts, appearing to the naked eye as a distinct dark brown spot. In the centre of this area there is a clear, refractile, circular spot, surrounded by four smaller dots equidistant

* This suggestion deals with the morphology of the carapace only in so far that it leaves out of account the many complex issues that might arise from a full consideration of the various minor grooves, other than the main cervical one, which are described by BOAS (*Decap. Slaegtskab., Vidensk. Selsk. Skr.*, 1880) as present on the carapace of various Decapods.

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from each other (fig. 2, *oc*). The appearance of these structures suggests at once that we have here to do with a group of ocelli. We cannot say positively, however, whether this is the case, since an attempt to ascertain by sectioning the structure of the organs was frustrated by the bad state of preservation of our material. There appears, however, to be no lenticular thickening, but rather a thinning away of the chitinous cuticle over the central spot, while the small lateral spots correspond to pits or actual perforations of the cuticle, and may possibly be merely the points of insertion of setæ which have been destroyed in the specimens examined. Whatever may be the nature of these or of the central spot, whether they be ocelli, as their appearance suggests, or not, we know of no structure in any adult Malacostracan with which they are comparable. Their position corresponds roughly with that of the "neck gland" of the Phyllopods, which has been identified with the embryonic "dorsal organ" of certain Malacostraca (*Edriophthalmata, Mysidæ*, &c.), but in the absence of further information as to their structure, it is impossible to guess what the meaning of these remarkable "ocelli" may be.

Antennules.—We have little to add to Mr THOMSON's description of these organs. We can confirm his account of the auditory organ which is lodged in the first joint of the antennule and opens to the exterior by a narrow slit on the upper surface near the anterior end of the joint (fig. 2, *e*). The possession of an auditory sac in this position has hitherto been regarded as characteristic of the Decapoda, and its presence in *Anaspides* is, therefore, a point of great interest. CLAUS has suggested, in connection with his discovery of paired otocysts in the head of certain *Amphipoda* (*Oxycephalus*), which he regards as probably homologous with the auditory organs of the *Decapoda*, that the possession of such organs in the region of the head was a character of the primitive *Malacostraca.** The discovery of the auditory organs of *Anaspides* certainly lends considerable support to this view. In the male the inner flagellum of the antennule is modified in a very remarkable manner. The proximal joints of the flagellum are swollen and distorted, and armed with stout curved serrated spines. While we are as yet completely ignorant of the habits of *Anaspides*, we may hazard the conjecture that this modification of the antennules in the male is suggestive of a prehensile function rather than the sensory one which Mr THOMSON attributes to them. A modification of the antennules for such a purpose, common enough among the *Entomostraca*, would be almost if not quite unique among *Malacostraca*.

Antenna.—The peduncle of the antenna presents only two joints below the origin of the scale or exopodite, agreeing in this respect with the antenna of *Euphausiidæ*, and differing from that of the *Mysidæ*, where three joints are present. The peduncle of the flagellum, or endopodite, is also remarkable in possessing only two joints (for we do not find the short third joint figured by Mr THOMSON) in place of the three joints usual among the *Malacostraca*.

* ". . . dass schon zu einer Zeit, in welcher die Edriophthalmen- und Podophthalmenzweige noch nicht gesondert waren, vor dem Gehirn ein bläschenförmiges Sinnesorgan gelegen war, auf welches wir das Gehörorgan von Decapoden und Oxycephaliden zurückzuführen hätten."—CLAUS, *Crustaceen-System*, p. 27.

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Mandible.—The body of the mandible (Plate I. fig. 3) is of the usual form, and carries a large three-jointed palp. The inner or cutting edge is divided into three regions, an anterior serrate (*a*), a posterior flattened (*c*) (the "molar process"), and a median (*b*), fringed with setæ, and separated from the anterior by a deep, and from the posterior by a shallow, notch. We have not only in the *Mysidæ* (Plate I. fig. 5) but throughout all the divisions of the Edriophthalmate Crustacea a somewhat similar three-fold division of the mandibular tomial edge, although the setæ fringing the median lobe in *Anaspides* are represented in these forms by a smaller number of comparatively large spines, while an accessory cutting blade (*lacinia mobilis* of HANSEN), unrepresented in *Anaspides*, is commonly developed. The mandible of the *Euphausiidæ* (Plate I. fig. 4), on the other hand, is very different and comparatively simple. The median division is absent, and the serrations of the anterior part merge with the posterior molar process. This type of mandible, which is regarded by BOAS * as the most primitive, seems more easily interpreted as a modification in the direction of simplification of the three-lobed type. This view is supported by SARS' † observation that the early larvæ of the *Euphausiidæ* have mandibles of a more complex type, resembling those of the *Mysidæ*. It seems probable, therefore, that the mandible of *Anaspides* belongs to a more primitive type than either the more complex organ of the *Mysidæ* or the apparently simple one of the *Euphausiidæ*.

The lower lip (labium or paragnatha) presents the general features of a bilobed plate with divergent lobes connected with each other for only a short distance at the base. It is chiefly remarkable for its large size, in which respect it even exceeds the exceptionally large dimensions of that organ in the *Euphausiidæ*. In connection with the large size of this organ, together with the apparent absence of an inner lobe associated with the first joint of the first maxillæ, it may be worth while to recall CLAUS' suggestion ‡ that the two halves of the labium (or paragnatha) represent the said missing portions of the maxillæ.

First Maxilla (Plate I. figs. 6 and 6*a*).—We are able to add some details of importance to the somewhat brief description of this organ given in the original paper. The two lobes (*lacinia interna* and *externa* of BOAS) are large, and directed somewhat forwards as well as inwards. The distal lobe is armed with stout toothed spines, and the proximal with feathered setæ. With the proximal lobe is connected an outwardly turned, rounded lobe (fig. 6, *ex*), not projecting beyond the outer edge of the maxilla, and lying closely adpressed to, though separate from, its posterior face. This outer lobe corresponds to the structure which in the first maxilla of other Schizopods (figs. 7 and 8) is usually regarded as the exopodite, but which HANSEN § regards as a process of the first joint of the limb. On the outer edge of the maxilla, somewhat on the anterior face, and about level with the division between the two

* *Stud. ü. d. Verwandschaftsbeziehungen d. Malakostraken. Morph. Jahrb.*, viii., 1882.

† *Challenger Rep.*, "Schizopoda," p. 153.

‡ *Neue Beiträge z. Morph. d. Crust. Arb. a. d. Zool. Inst. Wien*, vi., 1886, p. 15.

§ *Dijmphna Togtet Krebsdyr*, pp. 69–71. *Resumé*, p. 22.

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inner lobes, is placed a prominent, outwardly directed papilla (figs. 6 and 6*a*, *p*), which from its position is evidently a rudimentary palp. Two distinct articulations can be recognised in the basal part of the outer edge of the limb (fig. 6, 1 and 2), marking off two plates which correspond to those in the first maxilla of *Euphausiidæ* (Plate I. fig. 7, 1 and 2), identified by HANSEN (*l.c.*) as representing the first and second joints of the limb.

While agreeing in general structure with the corresponding appendage of *Euphausiidæ* (fig. 7), the first maxilla of *Anaspides* differs in the rudimentary state of the palp and in the small size of the "exopodite." In both of these characters it approaches the first maxilla of the *Mysidæ* (Plate I. fig. 8), in which the palp is absent, and the "exopodite" is reduced to a prominent ridge on the outer edge of the limb.

Second Maxilla (Plate I. fig. 9).—On the outer border of the limb, the exopodite or flabellum, which is so prominent a feature in other Schizopods, vanishes, though we may perhaps interpret as representing it that portion of the outer border which is fringed with short setæ. Apart from this difference, the structure of this limb is easily reducible to the type of the *Mysidæ* (Plate I. fig. 11), if we suppose the compression of the forwardly turned lobes to be carried so far that the lobes come to lie nearly at right angles to the plane of the limb. As in the *Mysidæ*, there are three lobes internal to the palp, the innermost being continued backwards, forming the straight inner edge of the maxilla. The palp, which in the *Mysidæ* consists of two joints, presents here, as in the *Euphausiidæ* (Plate I. fig. 10), only one. The palp and the outermost of the three lobes carry peculiar comb-like setæ, which have been figured by Mr THOMSON. The chitinous framework of the second maxilla presents a complex structure which we have not been able, from the material at our disposal, to elucidate completely. A point of some interest is the presence of a distinct segment corresponding to the outer or distal lobe, the fourth segment of HANSEN, which, according to that author, is absent in the second maxilla of *Mysis*.*

Thoracic limbs.—The thoracic limbs are remarkable for the powerful development of the endopodites, which are obviously more efficient walking-legs than those of most Schizopods. In this respect they approach the corresponding appendages of the *Lophogastridæ*.

The first thoracic limb or maxilliped (Plate II. fig. 12) presents a complicated and very peculiar structure. The first joint or coxa is very large, and carries on its external face two appendages, the branchial lamellæ (fig. 12, *ep*), each consisting of a very small basal joint and an elongated lanceolate compressed lamina. On its inner face the coxa is produced into *two* flattened setose lobes (fig. 12, *en*) (not *three*, as shown in THOMSON's figure). Of these the distal is the larger, expanding from a narrow neck to a somewhat triangular plate, and densely setose on the posterior face. The proximal is smaller, rounded, and fringed with setæ at the tip. Both lobes are articulated to the coxa in such a way as to allow a limited amount of movement,

* HANSEN, *Dijmphna Togtet Krebsdyr*, p. 70.

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and in the natural position they are turned forward so as to oppose the posterior face to the corresponding part on the other side. The second joint or basipodite is very small, and carries on its outer face the exopod, which in the limb under consideration is short, and indistinctly or not at all segmented. The third joint or ischium is long, compressed, and thinned away toward the inner edge, which is somewhat produced and fringed with setæ. The basipodite is followed by *five* distinct joints, so that the main axis of the limb presents one more than the normal number of seven joints. Such an increase in the number of joints is not found elsewhere in the case of the maxilliped, though the number seven may be exceeded in the posterior thoracic limbs from the third onwards in various *Mysidæ*. The last joint is very small, and bears a number (about four) of stout curved spines.

The succeeding limbs, while agreeing in general form with the maxillipeds, present certain modifications in detail. The second pair (Plate II. fig. 13) lack the peculiar internal lobes which in the maxillipeds are borne by the coxal joint. The branchial lamellæ are much broader, ovate in shape, and their points of attachment are close together on the outer face of the coxa. The latter is very small, and partly fused with the third joint, from which, however, it is marked off by a distinct line. The exopodite is very large, and consists of an unjointed peduncle bearing a many-jointed, setose flagellum. The ischium is cylindrical, and not produced inwards to a cutting edge, as in the maxilliped. The third, fourth, and fifth pairs of thoracic limbs agree precisely with the second, save that the line of demarcation between the second and third joints becomes less distinct as we go backwards, while in the sixth pair the fusion of these two joints is complete, and the exopod appears to spring from the ischium. In the seventh pair of thoracic legs the exopod is reduced to a short unjointed lobe, and the branchial lamellæ are reduced in size. In the last pair of thoracic legs all these appendages are wanting, and the limb is only represented by the endopodite.

As regards the branchial lamellæ or epipodites, the possession by each limb of two is a most striking and important feature. In no other adult Crustacean do we meet with such a character as these two separate epipodites. They are completely separate from one another, and in the maxilliped a wide interval occurs between their origins. THOMSON has suggested that these structures may be comparable to the epipodite of the *Euphausiidæ*, which, in the course of development, is at first bifurcate, but their arrangement in *Anaspides* does not suggest such a mode of origin, and it is important to note that on the maxilliped (where, in *Anaspides*, the two epipodites are most widely separate) the epipodite in the *Euphausiidæ* is always simple, it being branched only on the posterior limbs. There is, however, another possibility that suggests itself in connection with these epipodites. CLAUS has shown* that in the larvæ of certain *Penæidæ*, not only the podobranchiæ, but also the arthro- and pleurobranchiæ develop as outgrowths from the first joint of the legs, and he is inclined to regard the possession of three epipodial appendages as a primitive, if not primary, feature of the Malacostracan

* *Neue Beitr. z. Morph. d. Crust., Arb. Zool. Inst. Wien*, vi., 1886, pp. 42–45.

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thoracic limb. It is accordingly conceivable that we have in *Anaspides*, more nearly than in any other Malacostracan, the primitive gills in primitive number and position. The same very learned carcinologist maintains * that the brood-plates of the *Edriophthalmata*, and of the *Mysidæ*, as well as the branchial plates of the *Amphipoda*, spring, in their first development, not from the inner but from the outer side of the limb, and are in fact epipodites. And we venture to suggest the possibility, or even likelihood, of the double epipodites of *Anaspides* representing the gills and brood-plates of the *Amphipoda*, to which, in their ovate flattened form, they are not destitute of a certain resemblance.†

The presence of two movably articulated lobes springing from the inner face of the coxa in the maxillipeds is a feature which, so far as we know, is without a parallel among the Malacostraca. A single simple lobe is developed in the same position in the *Euphausiidæ*, but not in the *Mysidæ*. In the latter, the joint corresponding to the coxa is small or rudimentary, but an internal lobe or lacinia springs in their case from the enlarged second joint. Somewhat similar processes are developed from the second and third joints of the maxillipeds in the *Edriophthalmata*. It seems not improbable that the presence of two pairs of lateral appendages, internal lobes and epipodial lamellæ, springing from the coxal joint of the maxilliped, may indicate that this joint represents the fusion of two joints. HANSEN ‡ has given reasons for believing that the protopodite of the Crustacean limb is primarily three-jointed, not two-jointed as hitherto believed. The pre-coxal joint which he finds in *Nebalia*, and which, in the higher forms, is fused with the body, may possibly, in *Anaspides*, be fused with the coxal joint, and carry the proximal inner lobe of the maxilliped as well as the proximal epipodial lamella of that and the succeeding limbs.

Among minor features of interest in the thoracic legs of *Anaspides*, we note the reduction in size of the second joint, and its fusion with the third in the posterior members of the series. This joint is in the *Euphausiidæ* of much greater size, and in the *Mysidæ* attains, in the maxillipeds, an exaggerated development. Its fusion with the third joint has no parallel among the *Schizopoda*, but the same feature is found in the thoracic legs of many Decapods, especially in BOAS' group of *Reptantia*.

Another noteworthy point is the presence on the terminal joint of all the legs of a number (4–5) of stout curved claws. In the *Euphausiidæ* the terminal joint of the thoracic legs is tipped by a brush of setæ, while in most other Malacostraca the leg terminates in a single stout claw. *Anaspides* preserves for us a transitional stage between these two conditions, and it is interesting to note that, in the posterior thoracic legs, one of the claws exceeds the others very much in size, forming a further step in the direction of the "monodactyle" condition. With regard to the presence of eight joints in the thoracic legs, *Anaspides* may be compared with the Mysidan *Siriella*,

* *Op. cit.*, p. 34.

† The resemblance of the gill plates of *Anaspides* to those of *Amphipoda* is mentioned by Mr THOMSON in his preliminary paper, *l.c.*

‡ *Zool. Anz.*, xvi., 1893, p. 193.

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where the same number of joints is attained in the posterior thoracic legs. HANSEN, indeed, regards the possession of eight joints in the thoracic legs as characteristic not only of the *Mysidæ*, but also of the *Cumacea* and *Edriophthalmata*, while in the *Euphausiidæ* and *Decapoda* only seven joints are recognised. *Anaspides* further resembles the *Mysidæ* and the other groups above associated with them in having the legs flexed at the articulation between the fifth and sixth joints instead of between the fourth and fifth as is the case in the *Euphausiidæ* and *Decapoda*.

Abdominal Appendages.—The first five pairs of abdominal appendages have the exopodite developed as a long, many-jointed flagellum fringed with setæ, and forming a powerful swimming organ. The endopodite of the third, fourth, and fifth pairs is a small rounded lobe, two-jointed, but without any trace of *appendix interna*. The endopodites of the first and second pairs are in the female similar to those of the succeeding pairs, but in the male they are considerably modified. In the first pair the endopodite has the form of a thick lobe, curved inwards and hollowed on its anterior face, and having near the distal end of its inner edge a rudimentary *appendix interna* carrying a tuft of short "coupling hooks" or *retinacula*. In their natural position the two endopodites of the first pair are turned horizontally forwards between the bases of the last pair of thoracic legs; they are united by the retinacula, and together constitute, in consequence of their curved and hollowed form, a shallow trough in which the endopodites of the second pair play. The latter are longer, two-jointed, the long first joint carrying a bunch of *retinacula* and a few spines near the tip. These appendages lie coupled together within the groove formed by the first pair of endopodites, and between them and the sternal surface of the thorax. In short, these first and second abdominal appendages occupy much the same mutual position in the male *Anaspides* as they do in the male Crayfish. Among the Schizopods, the *Euphausiidæ* alone agree with *Anaspides* in the circumstance of the two anterior abdominal appendages being modified in the male, but no very close comparison is possible between the structures seen in *Anaspides* and the elaborate system of lobes, hooks, and stylets which differentiate these organs in the *Euphausiidæ*.

The sixth pair of abdominal appendages or uropods form with the telson a tail-fan of the characteristic podophthalmate form (Plate II. fig. 14). Both exopodite and endopodite have a slight median keel, and the exopodite is crossed a little beyond the middle of its length by a transverse suture which, starting from the outer edge, reaches to within a short distance of the inner edge.

The telson is less than half the length of the inner plates of the uropods, rounded at the end, which is fringed with spines.

SYSTEMATIC POSITION OF ANASPIDES.

If we are content to deal with the generally recognised limits and definitions of the Crustacean groups, and if we acknowledge the commonly but not uni-

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versally accepted association of *Euphausiidæ* with *Mysidæ* and their allies in the group *Schizopoda*, it is manifest, then, from the leading characters enumerated above, that *Anaspides* deserves to be also included in the *Schizopoda*, as it has been by its discoverer. But while we are content to place it so, its admission vastly increases the extent and importance of that group, for *Anaspides* stands far remote, within the group's now extended boundaries. The precise rank which we are to assign to it among the other members of the *Schizopoda* depends on the view we take as to the relations of these to one another. The commonly accepted classification of SARS * divides the sub-order *Schizopoda* into four families, *Lophogastridæ*, *Eucopiidæ*, *Euphausiidæ*, and *Mysidæ*, to which Mr THOMSON has added a fifth, *Anaspidæ*, for the reception of the genus under consideration. But the equivalence of these families is open to doubt, and some authorities have even denied the natural character of the group *Schizopoda* itself. BOAS † has proposed to break up the *Schizopoda* into two orders, *Mysidacea* (including the *Mysidæ*, *Lophogastridæ*, and *Eucopiidæ* of SARS) and *Euphausiacea* (*Euphausiidæ* of SARS), which he regards as not more nearly related to each other than they are respectively to the adjacent orders *Isopoda* and *Decapoda*. This dismemberment of the *Schizopoda*, which is also advocated by HANSEN, ‡ and adopted by ORTMANN, § has not been generally followed, but it must, we think, be recognised that it indicates the chief natural line of division among the organisms composing the group. This is the view taken by GERSTAECKER, ‖ who divides the *Schizopoda* into two tribes, *Holotropha* and *Hemitropha*, corresponding respectively to the orders *Mysidacea* and *Euphausiacea* of BOAS. Taking, then, the Mysid and the Euphausid as the two great types of *Schizopoda*, the points in which *Anaspides* approximates to one or other may be summarised as follows: the presence of only two joints in the peduncle of the antennæ, the characters of the first maxilla, the comparatively slight differentiation of the maxilliped from the rest of the thoracic limbs, the probable absence of a brood pouch, the presence of well-developed swimmerets in both sexes, and the modification of the first two pairs in the male, are all characters which incline towards the Euphausid type. With the exception of the first and last, we may look upon all these as points in which the Euphausid is more primitive and less differentiated than the Mysid type. On the other hand, the characters of the mandible and of the second maxilla resemble the Mysid rather than the Euphausid type, though not very closely comparable with either. The number of joints and mode of flexure of the thoracic legs are those of the *Mysidæ*, and we may also recall in this connection the fact that the carapace, the want of which is the most striking feature of *Anaspides*, is less strongly developed in the *Mysidæ*, where it leaves five of the thoracic segments free, than in the *Euphausiidæ*, where it is fused with all except the last. But besides the characters that permit us to acquiesce in its status as a Schizopod, the singular interest attaching to *Anaspides* is not diminished, and the difficulty of comprehending it is much increased by the coincident presence

* *Challenger Rep., Schizopoda.* † *Morph. Jahrb.*, viii., 1882. ‡ *Zool. Anz.*, xvi., 1893, pp. 202–5.
§ *Plankton Exp., Decapoden u. Schizopoden.* ‖ BRONN'S *Klassen u. Ordn. d. Thierreichs, Arthropoda.*

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of several important characters which point, as it were, in various directions towards presumably far distant branches of the Crustacean stem. Its auditory organs are those of a Decapod, the general form of the body is more that of an Amphipod than a Schizopod, while such characters as the double epipodial lamellæ, the coxal lobes of the maxilliped, and the remarkable dorsal "ocelli" (if such they be) are without parallel among the *Malacostraca*.

COMPARISON WITH PALÆOZOIC FORMS.

But *Anaspides* becomes still more striking in its interest when we pass from the limits of the recent fauna to seek comparisons with fossil forms. Among Palæozoic Crustacea there are several forms whose systematic position has hitherto been a complete puzzle to palæontologists. The genera associated together by PACKARD to form the groups *Syncarida* and *Gampsonychidæ* agree with each other and with *Anaspides*, and together with it stand apart from all other Crustacea whatsoever, in combining with the absence of a carapace, the presence of distinctly podophthalmate characters in antennules, antennæ, and tail-fan. These fossil forms had been compared by various palæontologists with the *Amphipoda*, the *Isopoda*, and the *Macrura*, while PACKARD emphasised more particularly their affinities with Schizopods. With this preliminary statement we pass to a detailed comparison of *Anaspides* with the Palæozoic genera.

PALÆOCARIS.

Palæocaris typus was first described in 1865, by MEEK and WORTHEN, from the Coal Measures of Illinois.* PACKARD, in 1886,† redescribed the species, and gave a restoration, which we reproduce (Plate II. fig. 15), certain characters being added from a later paper by the same author.‡ It is manifest from the figure that *Palæocaris* resembles *Anaspides* in general shape. The body is uniformly segmented, there being seven distinct segments in the thorax, while a carapace is completely lacking. The first five abdominal segments carry downwardly projecting pleura, the sixth segment being elongated and apparently cylindrical. The eyes are unknown, but the antennules have the three-jointed peduncle strongly developed and carrying a pair of flagella, of which the longer is one-third or one-half the length of the body. The antennæ have a broad scale or exopodite, extending nearly to the end of the peduncle, rounded at the end, and edged with setæ. Two joints of the peduncle are seen, and the flagellum is as long as the body. Of the thoracic appendages only six pairs are shown in the restoration, but it is stated in the text that the first two pairs in front of these are "gnathopods like those of existing Schizopods, especially *Petalophthalmus*," though

* *Proc. Acad. Nat. Sc. Philadelphia*, 1865; also, *Rep. Geol. Survey, Illinois*, iii., 1868.

† *Mem. Nat. Acad. Sci. Washington*, iii. (2), 1886. A preliminary abstract in *Amer. Naturalist*, xix., pp. 790–792, 1885.

‡ *Proc. Boston Soc. Nat. Hist.*, xxiv., pp. 211–213, 1889.

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the grounds for this statement are neither specified nor obvious. Each of the thoracic legs bears at its base an appendage about one-half as long as the limb itself, and nearly twice as broad, bluntly pointed at the tip, and indistinctly divided into three joints. PACKARD, in his preliminary communication, describes these as "breeding lamellæ," but in his later paper he withdraws this interpretation, and states that these appendages are to be regarded as "endopodites." What is meant by this use of the term endopodite we cannot understand, and the matter is made no clearer by PACKARD's reference to the "large ovate-lanceolate endopodites" of *Petalophthalmus*, for these can be no other than the brood-lamellæ themselves. In PACKARD's figures, what is apparently the main axis of the limb is lettered as the exopodite, but no explanation is given of this remarkable departure from ordinary nomenclature. The position of these so-called "endopodites" suggests that they correspond to the natatory exopodites of Schizopods, but they seem to differ very strikingly in shape from the slender, many-jointed exopodites usual in that group. An alternative hypothesis in regard to their homology is suggested by the resemblance of these organs in *Palæocaris* to the epipodial lamellæ of *Anaspides*, and more particularly to those of the maxilliped. This suggestion is founded partly on these considerations of size and appearance, further on the ground that a remarkable agreement has been shown to exist between *Palæocaris* and *Anaspides* in regard to other very important characters, and, lastly, on the ground that true exopods might very likely have perished in the fossil were they, as it is not unreasonable to suppose they were, similar either to those of the true Schizopods or of *Anaspides* itself; there can, indeed, be no strong presumption of their absence from *Palæocaris* in view of the many small and fragmentary structures that evidently lie entangled in this portion of the fossils. On this view, the single articulation at the base of each gill-lamella in *Anaspides* might conceivably be taken as the vestige of a segmentation formerly present in these organs, and partially preserved in *Palæocaris*.

The first five pairs of abdominal appendages in *Palæocaris* are powerful swimmerets, consisting each of a stout peduncle carrying "probably two slender rami," though only one is shown in the figure. The sixth pair form, with the telson, the usual tail-fan (fig. 15*a*). The outer plate or exopodite has a longitudinal median ridge, the homologue of which is to be recognised in *Anaspides*, and as in that form a transvere suture crosses the exopodite near the tip. The inner plate or endopodite is shorter than the outer, but longer than the telson, which is rounded at the tip and edged with setæ. PACKARD states that "there are traces of a pair of abdominal legs to each of the *seven* segments," but we may be permitted to doubt the existence of so extraordinary a deviation from the ordinary structure of the Malacostraca.

Particular interest attaches to the statement that the head of *Palæocaris* "is apparently composed of two segments." In the original figure of MEEK and WORTHEN there is no trace of this, but in PACKARD's restoration a line running downwards and backwards on the side of the cephalic region cuts off a wedge-shaped segment. The direction of this line is very different from that which we have called the cervical groove

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in *Anaspides*, but as it seems to have been only imperfectly visible on the fossil, it is possible that the restorer was guided to some extent in this point by the analogy of *Gampsonyx*, in which a similar line is very distinct.*

To sum up, *Palæocaris* resembles *Anaspides* closely in general form, in the number of free segments of the body, in the absence of a carapace, in the general characters of antennules and antennæ, of the swimmerets, and of the tail-fan. Points of doubtful agreement are found in the line which divides the two "segments" of the head, which may possibly represent the cervical groove of *Anaspides*, and in the so-called "endopodites" of the thoracic legs, which may correspond either to exopodites or to epipodial lamellæ, but which must be either one or other. Points of actual difference exist in the arrangement of the last-named organs, which, whatever their nature, are arranged in a single series, and are stated to be present on all the eight pairs of thoracic limbs, whereas in *Anaspides* both exopodites and gill-plates are absent from the last pair of legs. It is not improbable that some of these differences might be reduced, could the actual specimens of *Palæocaris* be re-examined in the light of the information afforded by *Anaspides*. PACKARD, throughout his papers, compares *Palæocaris* with the deep-sea *Petalophthalmus* and with no other Schizopod. Beyond the fact that the carapace in that very aberrant genus is unusually small, we do not see any indication of a special affinity between the two forms.

GAMPSONYX.

Gampsonyx fimbriatus, from the Coal Measures of Saarbrück,† was described by JORDAN and v. MEYER in 1854.‡ As will be seen from the figure (Plate II. fig. 17), it resembles *Palæocaris* in the segmentation of the body, but the line which in that genus divides the two "segments" of the head according to PACKARD is here more distinct, so that there appear to be eight free thoracic segments behind the head. It is very tempting to compare this with the condition found in *Anaspides*, but we have above given reasons for thinking that the so-called "first thoracic segment" in that form includes the region of the maxillæ as well as that of the maxillipeds. In the fossil form this does not appear to be the case, for the segment in question is narrower below than above, and would apparently correspond to only one pair of appendages.§ Still the coincidence in the number of apparent segments in the two forms is a very remarkable circumstance, and a re-examination of the fossils on this point is much to be desired.

The eyes of *Gampsonyx* are stated to be pedunculated. The antennules resemble those of *Palæocaris*. The antennæ have a large scale, rounded at the tip, a three-jointed peduncle, and a long flagellum.

* The *Palæocaris scoticus* of PEACH (*Trans. Roy. Soc. Edin.*, xxx., p. 85, pl. x. 3) differs very markedly from the American form in the arrangement of grooves on the head-region. It may possibly represent a distinct genus.

† These strata are now referred by some authorities to the Dyas.

‡ *Ueber d. Steinkohlenformation von Saarbrücken. Palæontographica*, iv., 1856.

§ JORDAN and v. MEYER remark that this segment varies slightly in length in different specimens.

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From the region of the first or second segment behind the head there springs a stout limb which reaches beyond the peduncle of the antennules. Four joints are visible, and the last two are armed with stout spines which, on the last joint, seem to form a sort of pseudo-chela. PACKARD identifies this limb as a mandibular palp, which he compares with the enormously developed palp of *Petalophthalmus*. Apart from the fact that the mandibular palp of the Malacostraca never presents more than three joints, the base of this limb appears to be placed much too far back to correspond with the position of the mandible, and it is more probable that it represents the first or second thoracic leg. With reference to the other thoracic legs, PACKARD says that these, "irrespective of the endopodites, are represented as biramous, and the two rami are drawn as of nearly equal length. It is probable that there has been a mistake in drawing the legs, as in none of the existing Schizopods, such as *Mysis* and its allies *Euphausia*, *Gnathophausia*, *Petalophthalmus* or *Chalaraspis*, are the legs thus thrice divided." In the preliminary note the same passage occurs, but "breeding lamellæ" is substituted for "endopodites," which makes the last sentence still more obscure. As a matter of fact, the legs are shown in JORDAN and v. MEYER'S figure as bifurcating *near the tip*, in the region, apparently, of about the third last joint. This appearance is doubtless due, however, to the overlapping of the appendages of the two sides. A confused mass which covers the bases of the legs shows traces of what may have been exopodites, and may possibly involve also the remains of gills.* The abdomen carries well-developed swimmerets. The tail-fan is similar to that of *Palæocaris*; the outer plates are divided by a suture, and the telson is comparatively short and rounded. It is stated that the telson appears to be fused with the last segment of the abdomen. If this be so, the tail-fan will bear some resemblance to that of certain *Isopoda*, particularly of the *Gnathiidæ*.

Gampsonyx, then, while agreeing in general characters with *Palæocaris*, differs from it in two points. It possesses an apparently free first thoracic segment only obscurely indicated, if at all, in *Palæocaris*, and it has a pair of powerful raptorial limbs identified by PACKARD as mandibular palps, but more probably to be referred to the first or second thoracic limbs. Neither of these differences increase the resemblance with *Anaspides*, unless indeed the "first thoracic" segments of the two are homologous, which is very doubtful. The character of the limbs indicates that *Gampsonyx* is probably a somewhat more specialised development from the common stock to which both *Anaspides* and *Palæocaris* are related.

ACANTHOTELSON.

Acanthotelson was described in 1860 by MEEK and WORTHEN from the Coal Measures of Illinois. It was doubtfully referred to the *Isopoda* by these authors, who, however, noted its resemblances to the *Decapoda*. PACKARD has given a restoration (Plate II.

* "Ueberdiess erkennt man dass an der Wurzel der vordern sieben oder acht Füsse noch besondere Anhängsel vorhanden waren, und zwar von namhafter Länge und Stärke."—JORDAN and v. MEYER, *op. cit.*

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fig. 16), and on his account alone the following remarks are based. In general shape *Acanthotelson* resembles the two genera which we have considered above, but the body appears to have been somewhat flattened dorso-ventrally. The head (fig. 16*a*) is stated by PACKARD to consist of two segments, but he continues:—"The second segment is distinctly separated by an impressed line from the first, but there is not a true articulation between them, so that the first and second cephalic segments may be said to be consolidated, and to represent the carapace of the *Schizopoda*." This account corresponds very closely indeed with the condition which we have described in *Anaspides*, although, as the line in question is not shown in the lateral view, we do not know whether its direction corresponds to that of the cervical groove in the last-named form. PACKARD describes the second segment as having "on each side a low, boss-like swelling, situated obliquely and prolonged in an oblique direction to the anterior outer edge." It is not impossible that the structure here referred to may represent the quadrilateral area which in *Anaspides* is cut off from the side of the cephalic region behind the cervical groove.

The eyes of *Acanthotelson* are unknown. The antennules have a three-jointed peduncle which carries two flagella. The peduncle of the antennæ is also three-jointed, and carries a moderately long flagellum. No antennal scales are shown in the figures, but traces of them are said to have been present, and in an allied form, placed by PACKARD in a separate genus, *Belotelson*, the antennal scales are said to be large, "resembling those of the *Macrura*."

Seven pairs of thoracic limbs are preserved, the first of these being very large, and armed with stout spines. The second pair are slightly enlarged, the following five pairs more slender and uniform in appearance. None of the thoracic limbs, so far as seen, are specialised as maxillipeds, and it is on this account, we presume, that PACKARD refers to them as "Schizopod-like," for no exopodites appear to have been observed.

The first five pairs of abdominal appendages in *Acanthotelson* are well-developed swimmerets, biramous, the exopodite being "lanceolate oval." PACKARD states that they resemble the corresponding appendages of *Squilla*, but elsewhere he states that they bear "a general resemblance to those of the Schizopoda."

The tail-fan is remarkable for the slender form of both branches of the uropods, as well as of the telson.

It will be seen from the above account that *Acanthotelson* resembles considerably the other two fossil forms which we have already considered. The spined raptorial limbs recall those of *Gampsonyx*, from which, indeed, the present form is mainly separated by the slender uropods and telson, and by the apparent absence of exopods from the thoracic limbs. We find no further hints of special affinity with *Anaspides*, save, perhaps, in the structure of the cephalic region, PACKARD'S description of which applies almost exactly to *Anaspides*.

PACKARD'S views as to the affinities of the above-mentioned genera are as follows:—

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In the first place, *Acanthotelson* and the doubtfully distinct *Belotelson*, are made to constitute a sub-order *Syncarida* "standing near or at the base of the Thoracostraca, not far from the *Stomapoda* and *Schizopoda*, and with appendages closely homologous with those of these two groups." We find it difficult to see how such an homology could be simultaneously close with the limbs both of Schizopod and Stomatopod. Under the name *Gampsonychidæ* PACKARD unites the genera *Palæocaris* and *Gampsonyx*, ranking them as a family of the Schizopoda. The reasons for this marked difference in the systematic rank assigned to these two groups are not very clear, for while PACKARD enumerates the resemblances between the *Syncarida* and the *Gampsonychidæ*, he nowhere indicates what he considers as the essential distinctions between the two.* As a matter of fact, *Acanthotelson* approaches very closely indeed to *Gampsonyx*; so far as we can gather, the only fundamental difference between the two lies in the fact that the thoracic limbs of *Acanthotelson* are stated not to possess exopods. In view, however, of the uncertain indications of these organs in *Gampsonyx* and in *Palæocaris* it seems hardly safe to lay too much stress on their alleged absence from *Acanthotelson*. By other palæontologists the genera in question have been generally referred, on account of the absence of a carapace, to the *Edriophthalmata*. *Gampsonyx* is described by JORDAN and v. MEYER as an Amphipod with characters of the *Decapoda*, particularly of the *Macrura*, while *Acanthotelson* was placed by its discoverers among the *Isopoda*, and these views have been recently endorsed, though with an expression of doubt, by so high an authority as Dr HENRY WOODWARD.†

We find, then, that *Anaspides* agrees with the extinct genera above enumerated in the essential point in which they have hitherto stood alone: the combination of Podophthalmate characters with a completely segmented body and the lack of a carapace. We have seen that some at least—probably all—of these genera show characters of the *Schizopoda*, to which group *Anaspides* is most closely allied. We find probable agreement in such points as the apparent division of the head region into two segments as by the "cervical groove" in *Anaspides*. Such differences as have appeared are readily explicable as comparatively unimportant differentiations which might be expected to occur within the limits of the group, or as due to the present imperfect state of our knowledge of the fossil forms. We conclude, therefore, that *Anaspides* is to be regarded as the representative of a group of primitive *Malacostraca*, which had already, in Palæozoic times, attained a certain degree of specialisation and a very wide distribution.

* Apparently, however, PACKARD'S views on this point have been somewhat modified, for in the 5th edition of his text-book of Zoology, published in the same year (1886) as the papers above referred to he uses the term *Syncarida* as including *all* the genera above named.

† "Anniversary Address," *Proc. Geol. Soc.*, lii., 1896.

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EXPLANATION OF PLATES.

Plate I.

Fig. 1. *Anaspides tasmaniæ*, G. M. Thomson. Male, enlarged 3½ diameters (from a photograph).

Fig. 2. Dorsal view of head region. *e*, opening of auditory sac; *o*, (supposed) "ocelli"; *c*, cervical groove; *a* and *b*, other grooves on integument (see text).

Fig. 3. Left mandible (palp omitted). *a*, *b*, and *c*, anterior, middle, and posterior (or molar) divisions of inner edge.

Fig. 4. *Boreophausia raschii*. Cutting edge of left mandible.

Fig. 5. *Mysis oculata*. do. do. *l.m*, *lacinia mobilis*.

Fig. 6. *Anaspides*. First maxilla of right side, posterior face.

Fig. 6*a*. Do. do. anterior face. *ex*, exopodite; *p*, palp; 1 and 2, first and second joints.

Fig. 7. *Boreophausia*. First maxilla of right side, posterior face.

Fig. 8. *Mysis*. do. do. do.

Fig. 9. *Anaspides*. Second maxilla of right side, posterior face. *p*, palp; 2, 3, and 4, lobes numbered according to Hansen.

Fig. 10. *Boreophausia*. Second maxilla of right side, posterior face. *ex*, exopodite: *p*, palp.

Fig. 11. *Mysis*. Second maxilla of right side, posterior face. *ex*, exopodite; *p*, palp.

Plate II.

Fig. 12. *Anaspides*. Maxilliped of left side, posterior face; *ex*, exopodite; *ep*, epipodial lamellæ; *en*, internal lobes of coxa.

Fig. 13. *Anaspides*. First walking leg of left side (second thoracic limb). *ex*, exopodite; *ep*, epipodial lamellæ.

Fig. 14. *Anaspides*. Tail-fan.

Fig. 15. *Palæocaris typus*, M. and W. Restoration (after Packard).

Fig. 15*a*. Do. do. Tail-fan (after Packard).

Fig. 16. *Acanthotelson stimpsoni*, M. and W. Restoration (after Packard).

Fig. 16*a*. Do. do. do. Dorsal view of head region, restored (after Packard).

Fig. 17. *Gampsonyx fimbriatus* (after Jordan and v. Meyer).

Note.—Since the above paper was in type, we have received from Mr G. M. Thomson (to whom we are much indebted for valuable material in connection with the study of Australasian Crustacea), specimens of *Anaspides* from a new locality, "Lake Field, a spot 40 miles from Hobart, Tasmania, at an elevation of about 4000 feet." This altitude corresponds with that of the pools on Mount Wellington, where the species was originally discovered by Mr Thomson.

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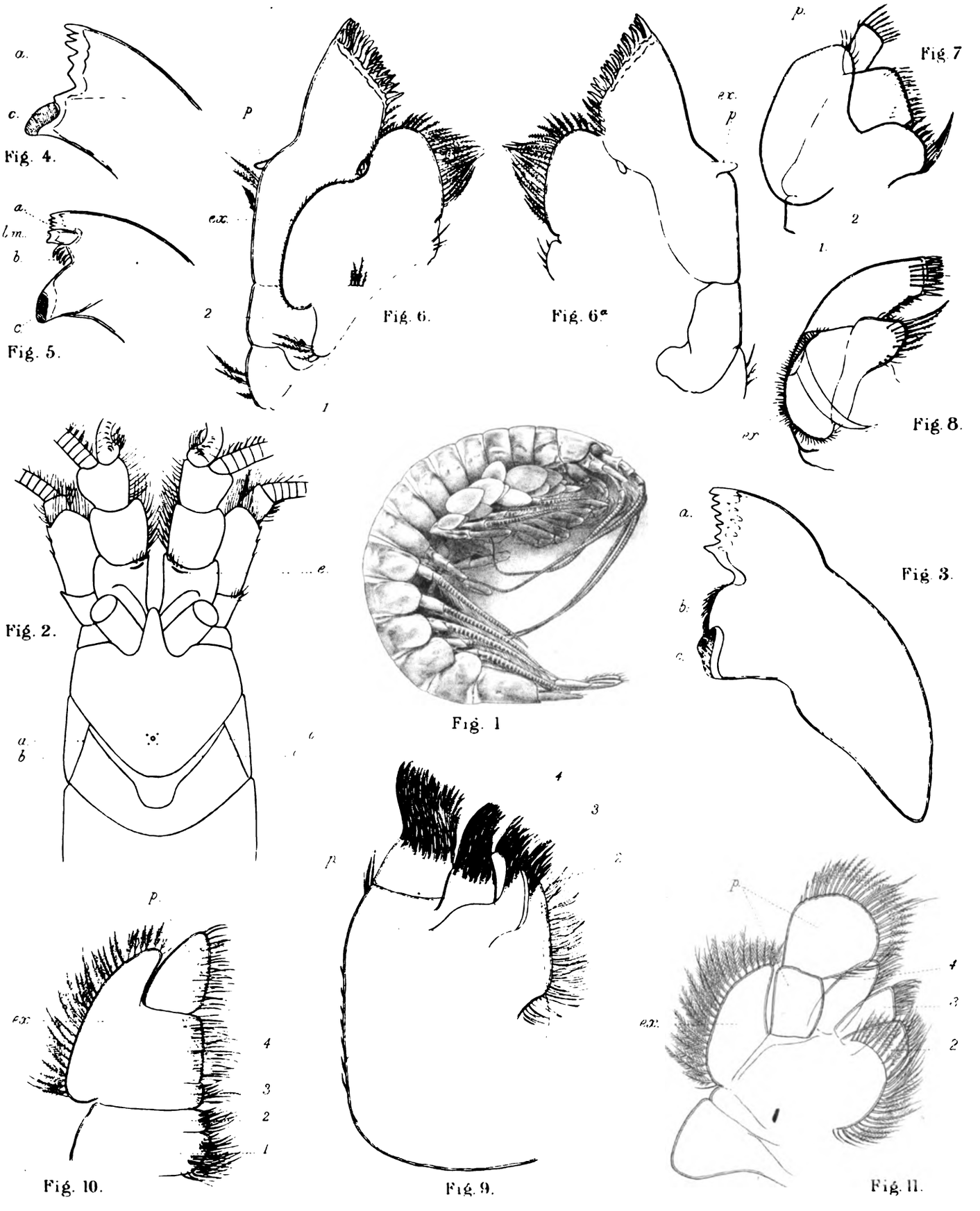
W. T. CALMAN ON THE GENUS ANASPIDES — PLATE I



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W. T. CALMAN ON THE GENUS ANASPIDES - PLATE II.

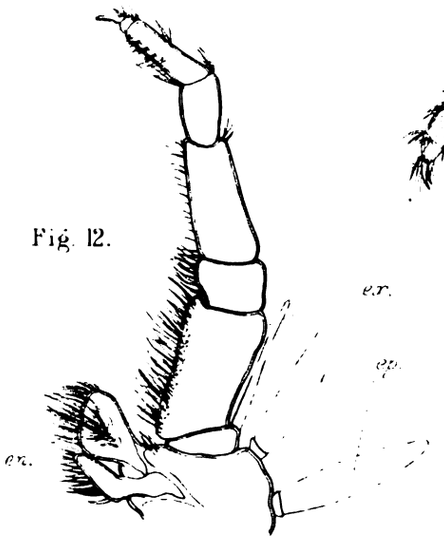

Fig. 12.

Fig. 13.

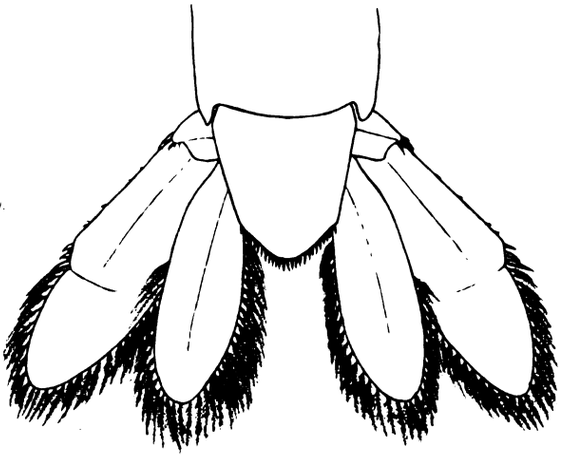

Fig. 14.

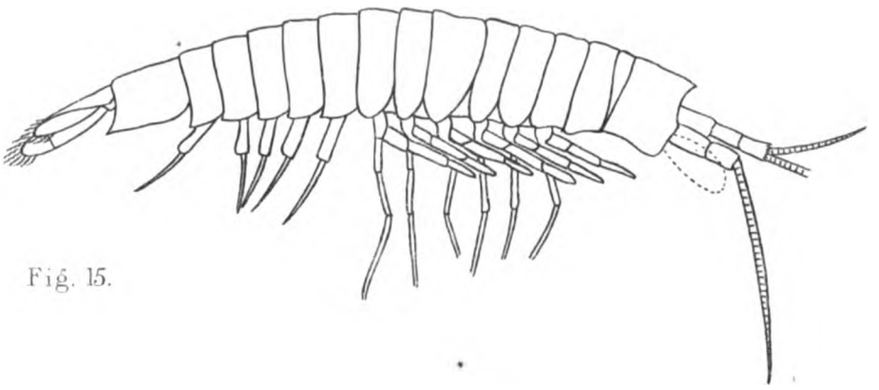

Fig. 15.

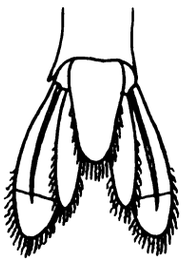

Fig. 15^a

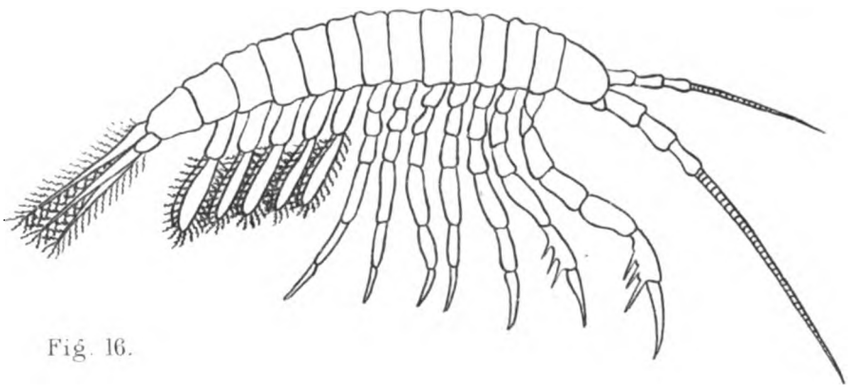

Fig. 16.

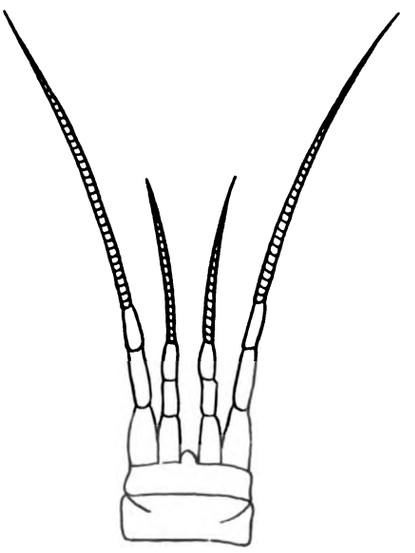

Fig. 16^a

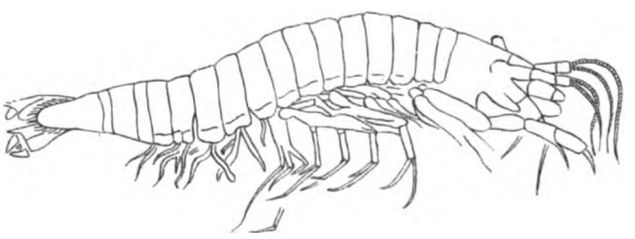

Fig. 17.

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XXIV.—*On the* p-*discriminant of a Differential Equation of the First Order, and on Certain Points in the General Theory of Envelopes connected therewith.* By Prof. CHRYSTAL.

(Read 15th June 1896.)

The theory of the singular solutions of differential equations of the first order, even in the interesting and suggestive form due to Professor CAYLEY (*Mess. Math.*, ii., 1872), as given in English text-books, is defective, inasmuch as it gives no indication as to what are normal and what are abnormal phenomena. Moreover, CAYLEY added an appendix to his theory regarding the circumstances under which a singular solution exists, which is misleading so far as the theory of differential equations is concerned, if not altogether erroneous.

The main purpose of the following notes is to throw light on the point last mentioned by means of a number of examples. I have also taken the opportunity to furnish simple demonstrations of several well-known theorems regarding the p-discriminant which do not find a place in the current English text-books.

It may be premised that in what follows we shall regard only such integral curves of the differential equation at any point as admit of an approximate representation of the form

$$y=\lambda x^{\alpha}+\mu x^{\beta}+\ldots\ldots$$

where α, β, ... are commensurable numbers, *i.e.*, only integrals which have an 'algebraic point' at x, y.

Nature of the Integral of the Equation—

$$A_0+A_1p+A_2p^2+\ldots\ldots+A_np^n=0 \quad . \quad . \quad . \quad . \quad (1)$$

at a Point on the p-*discriminant Locus, in the most General Case.*

We suppose that at the point in question A_0, A_1, ..., A_n are synectic: so that we have

$$\left.\begin{array}{l} A_0=a_0+b_0x+c_0y+d_0x^2+\ldots\ldots, \\ A_1=a_1+b_1x+c_1y+d_1x^2+\ldots\ldots, \\ A_2=a_2+b_2x+c_2y+d_2x^2+\ldots\ldots, \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ A_n=a_n+b_nx+c_ny+d_nx^2+\ldots\ldots \end{array}\right\} \quad . \quad . \quad . \quad . \quad (2).$$

We take the most general case, and suppose that two values of p, and no more, become equal at the point on the p-discriminant locus. Let this point be taken as

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origin; and let the x-axis be the tangent to the integral curve of (1). Then, corresponding to $x=0$, $y=0$, (1) must have two zero roots, and no more. Hence $a_0=0$, $a_1=0$, $a_2 \neq 0$. In the most general case the values of the other constants, b_0, c_0, &c., will be unrestricted, and the p-discriminant locus will not touch the x-axis.

If we confine ourselves to points infinitely near the origin, x will be an infinitely small quantity of the first order; y infinitely small, of second order at least; and p infinitely small, but not necessarily of the same order as x or y. (It will, in fact, be of the order $\alpha-1$, if α be the order of y, *i.e.*, of the same order as y/x.)

If we neglect in the equation (1) all quantities that are obviously not of the lowest order, we have in the most general case merely

$$b_0x+a_2p^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3);$$

the integral of which, subject to the condition $y=0$ when $x=0$, is

$$y=\pm\tfrac{2}{3}\left(-\frac{b_0}{a_2}\right)^{\frac{1}{2}}x^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

Hence the integral curve of (1) has a cusp at the origin (fig. 1).

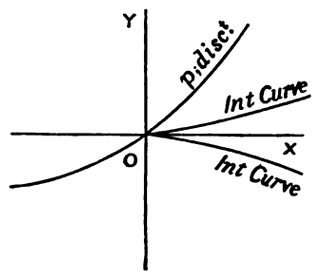

FIG. 1.

It follows that *the* p-*discriminant locus is in general the locus of cusps on the integral curves of the differential equation.*

On the Nature of the Integral at a Point on the p-*discriminant Locus where the Primitive in question touches that Locus.*

If we take the origin at the point in question and the tangent to the integral curve as x-axis, the differential equation will take the form

$$b_0x+c_0y+d_0x^2+(b_1x+c_1y)p+a_2p^2=0 \quad . \quad . \quad . \quad . \quad (1),$$

where all terms have been omitted which will obviously not be ultimately required for our approximations.

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The p-discriminant is given by combining (1) with

$$b_1x+c_1y+2a_2p=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

and regarding p as an arbitrary parameter For all points with which we are concerned the value of p is small.

Eliminating p, we have, therefore, for an approximation to the p-discriminant near the origin

$$b_0x+c_0y+d_0x^2-\frac{1}{4a_2}(b_1x+c_1y)^2=0 \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Hence the tangent to the p-discriminant at the origin is $b_0x+c_0y=0$.

The necessary and sufficient condition that the primitive touch the p-discriminant at the origin is therefore

$$b_0=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

The equation to the p-discriminant now reduces to

$$c_0y+d_0x^2-\frac{1}{4a_2}(b_1x+c_1y)^2=0 \quad . \quad . \quad . \quad . \quad . \quad (5),$$

wherein the term d_0x^2 cannot now be neglected. To the present order of approximation (5) may be written

$$\left.\begin{aligned} y&=\frac{b_1^2-4a_2d_0}{4a_2c_0}x^2 \\ &=kx^2, \text{ say.} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (6).$$

The differential equation reduces to

$$c_0y+d_0x^2+b_1xp+a_2p^2=0 \quad . \quad . \quad . \quad . \quad . \quad (7).$$

If we assume the approximation

$$y=\lambda x^a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

for the solution of (7), we get

$$a^2a_2\lambda^2x^{2a-2}+(ab_1+c_0)\lambda x^a+d_0x^2=0 \quad . \quad . \quad . \quad . \quad (9),$$

where $a_2 \neq 0$, and *in the general case* $d_0 \neq 0$.

Now $2a-2 >=< a$, according as $a >=< 2$. Hence, if $a<2$, we have $2a-2<a<2$; and, if $a>2$, $2a-2>a>2$. Since $a_2 \neq 0$, and $d_0 \neq 0$, neither of these hypotheses leads to an approximation.

We must therefore have $a=2$, and

$$4a_2\lambda^2+(2b_1+c_0)\lambda+d_0=0 \quad . \quad . \quad . \quad . \quad . \quad (10).$$

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In general (10) will have two distinct roots; and we conclude that *in general, when the primitive touches the* p-*discriminant two integral curves touch each other there, and we have a tac-point.*

If a branch of the integral curve touch the p-discriminant at *every* point of the latter, and *not merely at a single point*, as hitherto supposed, then the p-discriminant is a solution of (7), and (6) is a second approximation to a solution of (7). We must then have

$$4a_2k^2+(2b_1+c_0)k+d_0=0 \quad . \quad . \quad . \quad . \quad . \quad (11).$$

In other words, k is a root of (10). Hence one of the solutions furnished by (10) is the p-discriminant itself. The other, provided it be distinct, is a primitive touching the p-discriminant.

The condition (11) may be written (since $4a_2 \pm 0$,)

$$k^2+\frac{b_1}{2a_2}k+\frac{c_0}{4a_2}k+\frac{d_0}{4a_2}=0.$$

Now

$$c_0k+d_0=\frac{b_1^2-4a_2d_0}{4a_2}+d_0;$$

$$=\frac{b_1^2}{4a_2}$$

Hence (11) may be written

$$(k+b_1/4a_2)^2=0,$$

that is to say,

$$b_1^2-4a_2d_0+b_1c_0=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12).$$

This last condition reduces k to

$$k=-b_1/4a_2;$$

hence the other root of (10) is

$$\lambda=-d_0/b.$$

The approximations to the p-discriminant and the primitive, therefore, are

$$p\text{-discriminant, } y=-\frac{b_1}{4a_2}x^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (13);$$

$$\text{Primitive, } \quad y=-\frac{d_0}{b_1}x^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (14),$$

which are in general distinct. Hence, *if any branch of the* p-*discriminant is a solution of the differential equation that branch is an envelope singular solution; that is to say, at every point there is a primitive distinct from the branch of the* p-*discriminant in question which touches that branch.*

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We may note, in passing, that under the circumstances supposed the discriminant of the quadratic function

$$4a_2\lambda^2+(2b_1+c_0)\lambda+d_0$$

is a perfect square (since the roots are $-b_1/4a_2$ and $-d_0/b_1$). In fact, we have

$$\begin{aligned}(2b_1+c_0)^2-16a_2d_0 \\ &= 4(b_1^2-4a_2d_0)+4b_1c_0+c_0^2 \\ &= c_0^2,\end{aligned}$$

by virtue of (12).

To find the Conditions in the most General Case that the p-*discriminant furnish a tac-locus.*

The necessary conditions will be best seen by using NEWTON's diagram to estimate the relative orders of the terms in the differential equation. If we assume, as will be justified by the result, that it is unnecessary to go beyond the term in p^2, we may write our equation (with the same origin and axes as heretofore)

$$b_0x+c_0y+d_0x^2+e_0xy+f_0y^2+(b_1x+c_1y)p+a_2p^2=0 \quad . \quad . \quad . \quad (15).$$

If we remember that p is of the same order as y/x, we may, for the mere purpose of estimating the order of the terms, write the equation in the form

$$x^2(b_0x+c_0y+d_0x^2+e_0xy+f_0y^2)+xy(b_1x+c_1y)+a_2y^2=0 \quad . \quad . \quad (16).$$

Arranging the terms in NEWTON's diagram, we have figure 2. If two integral curves touch each other at the origin, we must have two approximations of the form $y=\lambda x^a$; and there must be a group of at least three effective terms: that is, there must be three outlying points in the diagram in a straight line next the origin. As a first condition, therefore, the term in x^3 must disappear, *i.e.*, $b_0=0$. If this condition alone were satisfied, we should fall back on the case last discussed. In order that the p-discriminant may not touch the two integral curves that have a common value of p at the origin, it is further necessary that $c_0=0$. If we retain only the three effective terms, the differential equation may be written

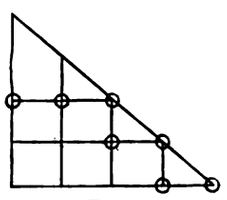

FIG. 2.

$$d_0x^2+b_1xp+a_2p^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (17).$$

If now we put $y=\lambda x^a$, we see that $a=2$, and

$$d_0+2b_1\lambda+4a_2\lambda^2=0.$$

There are therefore two first approximations, viz.:—

$$2a_2y=\{-b_1\pm\sqrt{(b_1^2-4a_2d_0)}\}x^2 \quad . \quad . \quad . \quad . \quad . \quad (18),$$

which correspond to two integral curves which touch at the origin.

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It remains to consider the nature of the p-discriminant near the origin. For this purpose it is necessary to retain more terms in the differential equation, for y and x are now of the same order of magnitude. Since p is small, the following equations will be sufficient for our purpose :—

$$\left.\begin{aligned} d_0x^2+e_0xy+f_0y^2+(b_1x+c_1y)p+a_2p^2=0, \\ b_1x+c_1y+2a_2p=0. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (19).$$

Eliminating p, we have for a first approximation to the p-discriminant

$$4a_2(d_0x^2+e_0xy+f_0y^2)-(b_1x+c_1y)^2=0 \quad . \quad . \quad . \quad . \quad (20).$$

This indicates that the p-discriminant has a double point at the origin. Hence, *the conditions that two integral curves touch each other and do not touch the* p-*discriminant at any particular point are* $b_0=0$, $c_0=0$; *and at such a point the* p-*discriminant has a double point.*

In order that the p-discriminant, or a branch of it, may be a tac-locus, the conditions $b_0=0$, $c_0=0$, must be satisfied at every point in question. Since it is impossible that every point of a continuous irreducible curve can be double, it follows that the p-discriminant must be reducible, must in fact contain a squared factor whose square root is the characteristic function of the tac-locus.

Hence, when the p-*discriminant furnishes a tac-locus, the two conditions* $b_0=0$ *and* $c_0=0$ *must be satisfied at every point of it. Its characteristic then contains a squared factor whose square root is the characteristic of the tac-locus.*

General expressions for the conditions for the existence of an envelope singular solution or a tac-locus can readily be obtained by transforming the differential equation

$$\phi(\mathrm{X},\mathrm{Y},\mathrm{P})\equiv \mathrm{A}_0+\mathrm{A}_1\mathrm{P}+\ldots\ldots+\mathrm{A}_n\mathrm{P}^n=0,$$

where the co-ordinates are (X, Y), and $\mathrm{P}=d\mathrm{Y}/d\mathrm{X}$, to the tangent and normal at the point (x, y) as axes. If the new co-ordinates be (ξ, η), $\varpi=d\eta/d\xi$, and $\lambda=1/\sqrt{(1+p^2)}$, $\mu=p/\sqrt{(1+p^2)}$, we have $\mathrm{X}=x+\lambda\xi-\mu\eta$, $\mathrm{Y}=y+\mu\xi+\lambda\eta$, and $\mathrm{P}=(p+\varpi)/(1-p\varpi)=p+(1+p^2)\varpi+p(1+p^2)\varpi^2+\ldots=p+\underline{\alpha\varpi+\beta\varpi^2}+\ldots$, say.

The differential equation then becomes

$$\phi(x+\lambda\xi-\mu\eta,\ y+\mu\xi+\lambda\eta,\ p+\alpha\varpi+\beta\varpi^2+\ldots\ldots)=0 \quad . \quad . \quad (21).$$

This gives to a sufficient approximation for our present purpose

$$\begin{aligned} &\phi+\phi_x(\lambda\xi-\mu\eta)+\phi_y(\mu\xi+\lambda\eta)+\phi_p(\alpha\varpi+\beta\varpi^2) \\ &+\tfrac{1}{2}[\phi_{xx}(\lambda\xi-\mu\eta)^2+\phi_{yy}(\mu\xi+\lambda\eta)^2+\phi_{pp}\alpha^2\varpi^2 \\ &+2\phi_{xy}(\lambda\xi-\mu\eta)(\mu\xi+\lambda\eta)+2\phi_{px}(\lambda\xi-\mu\eta)(\alpha\varpi+\beta\varpi^2) \\ &\qquad\qquad +2\phi_{py}(\mu\xi+\lambda\eta)(\alpha\varpi+\beta\varpi^2)]=0 \quad . \quad . \quad (22). \end{aligned}$$

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If we use our original notation for the coefficients, and remember that, since (x, y) is supposed to be a point on the p-discriminant, we have $\phi=0$, $\phi_p=0$, we get

$$\begin{aligned}
a_0&=\phi=0, & d_0&=\tfrac{1}{2}(\lambda^2\phi_{xx}+2\lambda\mu\phi_{xy}+\mu^2\phi_{yy}),\\
b_0&=\lambda\phi_x+\mu\phi_y, & e_0&=\lambda^2\phi_{xy}+\lambda\mu(\phi_{yy}-\phi_{xx})-\mu^2\phi_{xy},\\
c_0&=-\mu\phi_x+\lambda\phi_y, & f_0&=\tfrac{1}{2}(\lambda^2\phi_{yy}-2\lambda\mu\phi_{xy}+\mu^2\phi_{xx}),\\
a_1&=\alpha\phi_p=0, & a_2&=\tfrac{1}{2}(\beta\phi_p+\alpha^2\phi_{pp})=\tfrac{1}{2}\alpha^2\phi_{pp} \quad . \qquad (23).\\
b_1&=\alpha(\lambda\phi_{px}+\mu\phi_{py}), & & \\
c_1&=\alpha(-\mu\phi_{px}+\lambda\phi_{py}), & &
\end{aligned}$$

The general form of the condition $b_0=0$ is, therefore, $\lambda\phi_x+\mu\phi_y=0$, *i.e.*, $\phi_x+p\phi_y=0$. *Hence the necessary and sufficient condition in the most general case that the* p-*discriminant furnish an envelope singular solution is that the three equations in* x, y, p

$$\phi=0, \quad \phi_p=0, \quad \phi_x+p\phi_y{}^{*}=0 \quad . \quad . \quad . \quad . \quad . \qquad (24)$$

have a onefold infinity of common solutions. These common solutions furnish the envelope.

If, as will in general happen, the three equations have only a finite number of solutions in common, then the corresponding points will be tac-points at which the two touching branches touch the p-*discriminant locus.*

Since $b_0=0$, $c_0=0$ are equivalent to $(\lambda^2+\mu^2)\phi_x=0$, $(\lambda^2+\mu^2)\phi_y=0$, that is, to $\phi_x=0$, $\phi_y=0$, it follows that, *in the most general case, the* p-*discriminant furnishes a tac-locus when the four equations in* x, y, p

$$\phi=0, \quad \phi_p=0, \quad \phi_x=0, \quad \phi_y=0 \quad . \quad . \quad . \quad . \quad . \qquad (25)$$

have a onefold infinity of common solutions, the corresponding points being the tac-locus.

If, as may happen in special cases, the four equations have a finite number of common solutions, the corresponding points are tac-points, at which the touching integral branches do not in general touch the p-*discriminant, but at which the* p-*discriminant itself has double points.*

* It is easy to see in another way that $\phi_x+p\phi_y=0$ is simply the condition that the p-discriminant is an integral of the equation: for at any point of the p-discriminant $dx:dy$ is given by

$$\phi_x dx+\phi_y dy+\phi_p dp=0$$
$$\phi_{px}dx+\phi_{py}dy+\phi_{pp}dp=0.$$

Since $\phi_p=0$, the first of these equations gives

$$\phi_x+\phi_y dy/dx=0.$$

The condition that the p-discriminant be a solution is $p=dy/dx$; hence,

$$\phi_x+p\phi_y=0.$$

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In deducing the above conditions it has been tacitly supposed that p is not infinite. This special case may be treated by change of axes, by special investigation, or by the method of limits.

We have seen that the tac-locus, if it exists, is furnished by a squared factor in the p-discriminant. The first approximation,

$$4a_2(d_0x^2+c_0xy+f_0y^2)-(b_1x+e_1y)^2=0\,,$$

must therefore give a pair of coincident straight lines. The condition for this is

$$4a_2d_0f_0+e_0b_1c_1-a_2e_0^2-d_0c_1^2-f_0b_1^2=0\,.^*$$

This last equation ought, therefore, to be a derivative of $b_0=0$, $c_0=0$ when these are satisfied at every point of a branch of the p-discriminant. It will be an interesting test of the accuracy of the foregoing theory to verify that this is actually the case. If we substitute the values of a_2, &c., calculated above, we find

$$\begin{aligned}&4a_2d_0f_0+e_0b_1c_1-a_2e_0^2-d_0c_1^2-f_0b_1^2\\&\equiv\tfrac{1}{2}a^2(\lambda^2+\mu^2)^2\{\phi_{pp}(\phi_{zz}\phi_{yy}-\phi^2_{zy})-\phi_{zz}\phi^2_{py}+2\phi_{zy}\phi_{pz}\phi_{py}-\phi_{yy}\phi^2_{pz}\}\,,\\&\equiv\tfrac{1}{2}(1+p^2)^2\{\phi_{pp}(\phi_{zz}\phi_{yy}-\phi^2_{zy})-\phi_{zz}\phi^2_{py}+2\phi_{zy}\phi_{pz}\phi_{py}-\phi_{yy}\phi^2_{pz}\}\,.\end{aligned}$$

Now, from the equations $\phi_p=0$, $\phi_z=0$, $\phi_y=0$, which are supposed to be satisfied at every point of the branch of the p-discriminant in question, we have

$$\begin{aligned}\phi_{pz}dx+\phi_{py}dy+\phi_{pp}dp&=0\,,\\\phi_{zz}dx+\phi_{zy}dy+\phi_{pz}dp&=0\,,\\\phi_{zy}dx+\phi_{yy}dy+\phi_{py}dp&=0\,;\end{aligned}$$

whence, eliminating $dx:dy:dp$, we find

$$\phi_{pp}(\phi_{zz}\phi_{yy}-\phi^2_{zy})-\phi_{zz}\phi^2_{py}+2\phi_{zy}\phi_{pz}\phi_{py}-\phi_{yy}\phi^2_{pz}=0\,,$$

which establishes the derivation.

Geometrical Interpretation of the Conditions for a tac-locus.

If we regard (x, y, p) as the co-ordinates of a point in space of three dimensions, then the equation $\phi(x, y, p)=0$ may be taken as representing a surface. *The conditions for a tac-point or a tac-locus are therefore simply that the surface $\phi=0$ have a conical point or a double line.*

Looking at the matter from this point of view, or considering the symmetry of the conditions as regards (x, y, p), we see at once that *if the differential equation*

* It may be of interest to note that this is the condition that the function

$$a_2p^2+(b_1x+c_1y)p+d_0x^2+e_0xy+f_0y^2,$$

obtained by retaining only such terms of the characteristic of the differential equation as are required to determine an accurate first approximation to the p-discriminant, is decomposable into factors which are integral and linear in x, y, p.

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$$\phi(x, y, p) = 0$$

have a tac-locus, then the five equations, $\phi(x, p, y) = 0$, $\phi(y, x, p) = 0$, $\phi(y, p, x) = 0$, $\phi(p, y, x) = 0$, $\phi(p, x, y) = 0$, *have each in general a tac-locus also.*

We say, *in general*, because it may happen that there is not in each case a onefold infinity of values of x and y, and then we have merely a tac-point. Thus, for example, let us take GLAISHER'S Example VIII. (*Math. Mess.*, xii. p. 6)

$$\phi \equiv (x^2 - a^2)p^2 - 2xyp - x^2 = 0.$$

The conditions for a tac-locus are $(x^2 - a^2)p^2 - 2xyp - x^2 = 0$, $2xp^2 - 2yp = 0$, $-2xp = 0$, $2(x^2 - a^2)p - 2xy = 0$. These are satisfied by $x = 0$, $y = y$, $p = 0$: so that $x = 0$ is a tac-locus. If we interchange x and y, we get the equation

$$(y^2 - a^2)p^2 - 2xyp - y^2 = 0.$$

To get the tac-conditions for this new equation we have to adjoin the equations $x = x$, $y = 0$, $p = 0$: $y = 0$ is, therefore, a tac-locus. If, however, we interchange p and y, we get

$$(x^2 - a^2)y^2 - 2xyp - x^2 = 0,$$

the additional tac-conditions for which are $x = 0$, $y = 0$, $p = p$. Here, therefore, there is merely a tac-point.

Since a surface of the second degree cannot have a double line without degenerating, it follows that *an irreducible differential equation of the first order which is integral in* x, y, p, *and whose degree in* (x, y, p) *collectively does not exceed the second, can have no tac-locus*, a result which might also have been deduced from the remark made above, p. 810.

Locus of the Points of Inflexion on the Integral Curves of the Differential Equation—

$$A_0 + A_1 p + \ldots\ldots + A_n p^n = 0.$$

If, instead of considering the integral curve as a locus of points, we consider it as the envelope of its tangents, we see that to a locus of cusps on the integral curves corresponds a locus of inflexions. As the cusp-locus is present in the general case, the inflexion-locus will also be present in the general case.

The conditions for an inflexion on one of the integral curves at any point can readily be found by the methods above used. Let us take, as before, the point in question for origin and the tangent to the integral in question as axis of x; then, remembering that y is of higher order than x, and omitting terms that are *prima facie* negligible, we may write the differential equation in the form

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$$(b_0x+c_0y+d_0x^2)+a_1p=0\,, \quad . \quad . \quad . \quad . \quad . \quad . \quad (26),$$

where $a_1 \neq 0$, since only one value of $p=0$ when $x=0$, $y=0$.

For the purpose of estimating the order of the terms we may write our equation in the form

$$x(b_0x+c_0y+d_0x^2)+a_1y=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (27),$$

the NEWTON's diagram for which is given by the annexed figure (3). Hence, if $b_0 \neq 0$, the approximation to the integral curve is given by $b_0x+a_1p=0$, which gives $2a_1y+b_0x^2=0$, corresponding to an ordinary point. As a necessary condition, we must therefore have $b_0=0$; and this is, in general, sufficient; for the first approximation is now given by $d_0x^2+a_1p=0$, which gives $3a_1y+d_0x^3=0$, corresponding to an inflexion. If we translate this result into general symbols, we have as the condition for an inflexion at (x, y), on the branch corresponding to the value p, $\phi_x+p\phi_y=0$. *Hence the locus of points which are inflexions on integral curves is given by*

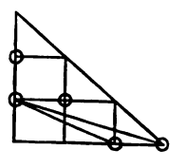

FIG. 3.

$$\phi=0,\ \phi_x+p\phi_y=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28),$$

where p *is to be treated as an arbitrary parameter, viz., it is the tangent of the inclination to the* x*-axis of that branch of the integral curve which has an inflexion at* (x, y).

Comparing the conditions for an envelope singular solution with the result now obtained we see that they may be expressed as follows:—*In order that there may be an envelope singular solution it is necessary and in general sufficient that the cusp-locus and the inflexion-locus have a branch in common.*

Since the differential equation to the orthogonal trajectories of the family represented by

$$\phi(x,y,p)=0\,,$$

is

$$\phi(x,y,-1/\varpi)=0\,,$$

the *inflexion-locus for the orthogonal trajectories is given by*

$$\left.\begin{array}{l}\phi(x,y,-1/\varpi)=0\,,\\ \phi_x(x,y,-1/\varpi)+\varpi\phi_y(x,y,-1/\varpi)=0\end{array}\right\};$$

or, putting $p=-1/\varpi$, by

$$\phi(x,y,p)=0, p\phi_x(x,y,p)-\phi_y(x,y,p)=0.$$

Since, in general, $\phi_x+p\phi_y=0$, $p\phi_x-\phi_y=0$ are equivalent to $\phi_x=0$, $\phi_y=0$, and since the p-discriminant locus is obviously the same for the original family and for the orthogonal trajectories, it follows that *at a tac-point where the tangent does not touch the* p*-discriminant locus this locus must have a point in common with the inflexion-loci of*

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the original family and of the orthogonal trajectories. The condition for the existence of a tac-locus where there is no contact with the p-*discriminant locus is that the* p-*discriminant locus, the locus of inflexions, and the locus of inflexions on the orthogonal trajectories must have a branch in common.**

Discussion of the Equation

$$c_0y+d_0x^2+b_1xp+a_2p^2=0.$$

Since this equation gives the first approximation in the general case to the form of an integral curve in the neighbourhood of an envelope, it is a matter of some interest to investigate the general nature of its integral.

Since $a_2 \neq 0$, we may put $a_2=1$. Dropping the suffixes we may write

$$ay+bx^2+cxp+p^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (29).$$

When the Equation (15) *has an Envelope Singular Solution its Integral is a Family of Algebraic Curves* (*Parabolæ*).

If we put $\xi=x^2$, $\varpi=dy/d\xi$, (29) becomes

$$ay+(b+2c\varpi+4\varpi^2)\xi=0.$$

If we solve for ϖ and put $y=v\xi$, we get

$$v+\xi\frac{dv}{d\xi}=\frac{-c\pm\sqrt{(c^2-4b-4av)}}{4}:$$

whence

$$\frac{4adv}{c+4v\pm\sqrt{(c^2-4b-4av)}}+\frac{d\xi}{\xi}=0.$$

Let $u=\pm\sqrt{(c^2-4b-4av)}$: then we have

$$\frac{-2udu}{ac+c^2-4b-u^2-au}+\frac{d\xi}{\xi}=0.$$

* The general existence of the cusp-locus of the integral family of a differential equation of the first order was indicated as early as 1851 by De Morgan (*Camb. Phil. Trans.*, vol. ix. pt. 2, p. 113). The earliest absolutely explicit statement of the theorem seems to have been made by Darboux (*Comptes Rendus*, t. lxx. p. 1331; also t. lxxi. p. 267, 1870). In an extremely interesting paper in the *Bulletin des Sciences Mathematiques*, &c., t. iv., 1873, p. 158, Darboux establishes most of the propositions above given. It is surprising that Darboux's work does not seem to have attracted the notice of Cayley. Reference may also be made to Clebsch, *Mathematische Annalen*, Bd. vi. p. 211, 1873; and to Clebsch's theory of "Connexes," *Vorlesungen uber Geometrie*, Bd. i. p. 1014 *et seqq.* We have thought it worth while to deduce these results throughout by the approximative method first employed by Briot and Bouquet, because this method is a general one, applicable to the discrimination of special cases, such as arise when an envelope is also a cusp-locus or a tac-locus, &c.; and because this method is little used by English mathematicians.

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Now, by (12), since (15) has a singular solution we must have

$$ac+c^2-4b=0.$$

Hence

$$\frac{2du}{u\pm a}+\frac{d\xi}{\xi}=0.$$

The integral is therefore

$$(u\pm a)^2\xi=\mathrm{A},$$

or

$$ax\pm\sqrt{(-acx^2-4ay)}=\mathrm{B},$$

the rational form of which is

$$(ax-\mathrm{B})^2+a(4y+cx^2)=0 \quad . \quad . \quad . \quad . \quad . \quad (30),$$

representing a family of parabolæ the envelope of which is obviously the parabola

$$4y+cx^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31).$$

To find the Condition that the Equation (15) *may have an Algebraic Integral*

$$(a\neq 0,\quad b\neq 0,\quad c\neq 0).$$

Proceeding as before, we reduce the equation to the form

$$\frac{2udu}{u^2+au-f}+\frac{d\xi}{\xi}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (32),$$

where

$$f=c^2+ac-4b \quad . \quad . \quad . \quad . \quad . \quad . \quad (33).$$

Let

$$u^2+au-f\equiv(u-\lambda)(u-\mu).$$

Then

$$\frac{2u}{u^2+au-f}=\frac{2}{\lambda-\mu}\left(\frac{\lambda}{u-\lambda}-\frac{\mu}{u-\mu}\right);$$

and the integral of (6) is

$$\frac{2\lambda}{\lambda-\mu}\log(u-\lambda)-\frac{2u}{\lambda-\mu}\log(u-\mu)+\log\xi=\mathrm{C} \quad . \quad . \quad . \quad (34).$$

If

$$\beta=a^2+4f=a^2+4c^2+4ac-16b,$$
$$=(a+2c)^2-16b, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35),$$

then

$$2\lambda=-a+\sqrt{\beta},\quad 2\mu=-a-\sqrt{\beta};$$

and we get for the primitive

$$\left(\frac{-a}{\sqrt{\beta}}+1\right)\log(u-\lambda)-\left(\frac{-a}{\sqrt{\beta}}-1\right)\log(u-\mu)+\log\xi=\mathrm{C};$$

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or

$$\left(\frac{u-\mu}{u-\lambda}\right)^{\frac{a}{\sqrt{\beta}}}(u-\lambda)(u-\mu)\xi=A \quad . \quad . \quad . \quad . \quad . \quad (36),$$

where

$$u=\mp\sqrt{\{c^2-4b-4ay/x^2\}}.$$

Since $a\neq 0$, it is obvious that the necessary and sufficient condition that (36) be algebraic is that β be a perfect square, that is, that $(a+2c)^2-16b$ be a perfect square. It is assumed, of course, that a, b, c are commensurable.

The condition that (29) may have a singular solution is

$$c^2+ac-4b=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (37).$$

This is not in general satisfied when the primitive is algebraic; although the primitive is always algebraic when (37) is satisfied; viz., in this case, $(a+2c)^2-16b$ reduces to $(a+2c)^2-4c^2-4ac=a^2$.

As this contradicts a well-known result of CAYLEY'S, it may be well to examine a simple particular case of a differential equation which has an algebraic primitive, but has no singular solution. By means of the above results we can construct an infinity of such cases; the fact being that it is the exception and not the rule that there is a singular solution when the primitive is algebraic.

Example of a Differential Equation which has an Algebraic Primitive but has no Singular Solution.

Consider

$$3y+\tfrac{1}{3}x^2-\tfrac{2}{3}xp+p^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

the p-discriminant of which is

$$y=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2),$$

and gives no solution of (1).

We may write (1) in the form

$$p=\tfrac{1}{3}x\pm\sqrt{(-3y)},$$

or, if $x^2=\xi$,

$$2\frac{dy}{d\xi}=\tfrac{1}{3}\pm\sqrt{\left(-\frac{3y}{\xi}\right)}.$$

Put $y=v\xi$, and we get

$$2\xi\frac{dv}{d\xi}+2v=\tfrac{1}{3}\pm\sqrt{(-3v)},$$

which may be written

$$\frac{4dv}{4v-\frac{2}{3}\mp\sqrt{(-12v)}}+\frac{d\xi}{\xi}=0 \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Let

$$u=\mp\sqrt{(-12v)},$$
$$u^2=-12v;$$

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whence

$$\frac{2udu}{u^2+2-3u}+\frac{d\xi}{\xi}=0;$$

that is,

$$\left(\frac{4}{u-2}-\frac{2}{u-1}\right)du+\frac{d\xi}{\xi}=0 \quad . \quad . \quad . \quad . \quad . \quad (4).$$

The integral of (4) is

$$(u-2)^2\sqrt{\xi}/(u-1)=-4c \quad . \quad . \quad . \quad . \quad . \quad (5).$$

Now (5) may be written

$$(u^2-4u+4)x+4cu-4c=0,$$

or

$$xu^2-4(x-c)u+4(x-c)=0.$$

Hence

$$u=2\frac{(x-c)\pm\sqrt{\{c(c-x)\}}}{x};$$

whence, squaring and substituting the value of u, we get

$$3y=-(x-c)(x-2c)\pm 2c^{\frac{1}{2}}(c-x)^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad (6);$$

or, in rational form,

$$(x^2+12y)c^2-2x(x^2+9y)c+(x^2+3y)^2=0 \quad . \quad . \quad . \quad . \quad (7).$$

The c-discriminant is therefore

$$x^2(x^2+9y)^2-(x^2+12y)(x^2+3y)^2=0,$$

which reduces to

$$y^3=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

The primitive curve can be readily traced from (6).

We observe, in the first place, that the value of y is unchanged if we change the signs of both c and x. It follows that the curve of the family for any negative value of c is the image in the y-axis of the curve for the corresponding positive value of c. We may therefore confine our attention to positive values of c; and we also see that, for real values of y, x must not exceed $+c$.

Since $2c-x \nless 2\sqrt{\{c(c-x)\}}$, the value of y is negative for all admissible values of x, and vanishes only when $x=0$ and $x=c$.

From (6) we have

$$y'=\tfrac{1}{3}(3c-2x)\mp\sqrt{\{c(c-x)\}} \quad . \quad . \quad . \quad . \quad . \quad (9);$$

$$y''=-\tfrac{2}{3}\pm\tfrac{1}{2}\sqrt{\{c/(c-x)\}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10).$$

If we speak of the parts of the curve corresponding to the upper and lower signs as the first and second branches, we see at once that for the second branch y' is always positive and y'' always negative. This branch crosses the y-axis at $(0, -\frac{4}{3}c^2)$ and is uniformly convex to the x-axis.

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For the first branch y' vanishes when $x=0$ and when $x=\frac{3}{4}c$; these give a maximum turning-point at the origin and a minimum turning-point at $(\frac{3}{4}c, -\frac{7}{24}c^2)$.

At $x=c$ there is obviously a cusp, the value of y' being $\frac{1}{3}c$.

Near the origin, approximation to the first branch is given by

$$y=-\tfrac{1}{12}x^2+\tfrac{1}{24}\frac{x^3}{c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11);$$

from which we see that all the first branches osculate the parabola $12y+x^2=0$ at the origin; and depart less and less from it the more we increase c.

It is also obvious from (7) that this parabola

$$12y+x^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

is the limiting form of the integral curve corresponding to $c=\infty$; and also that it divides the region below the x-axis into two districts, in the upper of which the two values of c corresponding to the two curves of the family which pass through a given point are of the same sign in the lower of which of opposite signs.

The following figure, therefore, gives a sufficient representation of the forms of the curves of the family for positive values of c. For negative values we have simply to reflect the diagram in the y-axis. KOAB, figure (4), is the curve corresponding to any

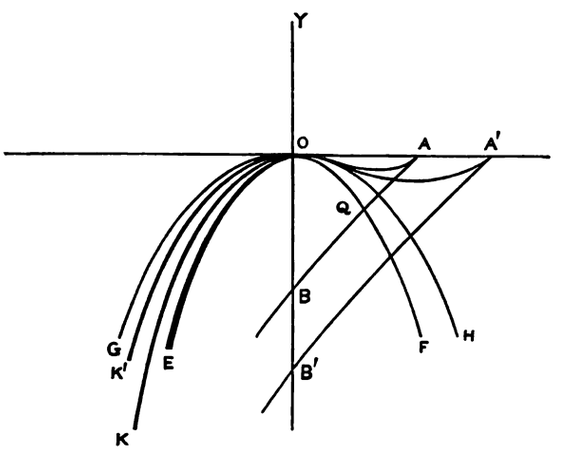

FIG. 4.

finite value of c, K'OA'B' that corresponding to a greater value. The parabola GOH is the limiting form corresponding to $c=\infty$, the second branch of which is altogether at ∞. The limiting form corresponding to $c=+0$ is the left-hand part of the parabola EOF, whose equation, as is easily seen from (7), is

$$3y+x^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13).$$

We have thickened this part to indicate that it must be reckoned twice; for, as A

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moves towards O, it is obvious that OK approaches OE from one side and AB approaches the same limit from the other.

It must be carefully noticed that the other half, OF, of the parabola (12) is the limiting form for negative values of c. If this be forgotten, confusion will arise regarding the two branches of the primitive family which pass through any point of the plane.

Thus, for example, the two curves through P are a first branch of one primitive and a second of another, both corresponding to positive values of c. At any point Q below EOF the two curves are both second branches—one corresponding to a positive, the other to a negative value of c: in particular, at a point Q on the dotted branch of (13) the two curves are BQA and OQF.

At the origin all the primitive curves have the same first approximation, viz., the parabola (12). The second approximations for any two curves c and c' are, as we have seen,

$$\left.\begin{array}{l} x^2+12y-\frac{1}{2}x^3/c=0 \\ x^2+12y-\frac{1}{2}x^3/c'=0 \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14).$$

Quà intersections at the origin this pair of equations is equivalent to $x^2+12y=0$ and $x^3=0$, that is to $x=0$, $y=0$ thrice. Hence every integral curve is intersected by its consecutive in three points at (0, 0).

It is readily found from NEWTON's diagram that all the integral curves have at ∞ the common first approximation

$$(x^2+3y)^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15);$$

and that the second approximation to any curve c is

$$(x^2+3y)^2+4cx^3=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16).$$

If we apply to (16) the linear transformation $x=\xi/\eta$, $y=1/\eta$, we get

$$(\xi^2+3\eta)^2+4c\xi^3\eta=0,$$

or, to the same approximation as before,

$$(\xi^2+3\eta)^2-\tfrac{4}{3}c\xi^5=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17).$$

To the multiple point (0, ∞) on (16) corresponds the multiple point (0, 0) of the same order and species on (17).

Now (17) has a node-cusp at (0, 0), for which $\delta=1$, $\kappa=1$ (see SALMON's *Higher Plane Curves*, § 243).

If we combine with (17) another equation

$$(\xi^2+3\eta)^2-\tfrac{4}{3}c'\xi^5=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18),$$

and confine ourselves to intersections at the origin, we shall find the multiplicity of the intersection of any integral curve with another consecutive or non-consecutive at (0, ∞).

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Now (17) and (18) are equivalent to $(\xi^2+3\eta)^2-\frac{4}{3}c\xi^5=0$, $\xi^5=0$; that is to $(\xi^2+3\eta)^2=0$, $\xi^5=0$; that is to $\eta^2=0$, $\xi^5=0$. Hence ten intersections of (17) and (18) are condensed at (0, 0); and therefore any two consecutive (or non-consecutive) integral curves intersect ten times at $(0, \infty)$ . . .

These results may be verified by finding the x-eliminant of (7) and the equation derived by differentiating its characteristic with respect to c, viz.,

$$(x^2+12y)c-x(x^2+9y)=0 \quad . \quad . \quad . \quad . \quad . \quad (19).$$

The result is

$$x^3(x-c)^3=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (20).$$

Since the degree of (20) ought to be 16, we see that two consecutive integral curves intersect in 3 points at (0, 0), 3 at $(c, 0)$, and 10 at $(0, \infty)$.

The family of integral curves has therefore no envelope, real or imaginary; but merely the cusp-locus $y=0$ and the fixed tac-points (0, 0), $(0, \infty)$.

It appears, therefore, that the p-discriminant curve in the present case, notwithstanding that the primitive is algebraic, is, as usual, a cusp-locus, and is in no proper sense an envelope. It might, perhaps, be contended that it is an envelope in the sense that every primitive touches every other at one particular point, viz., (0, 0), or $y=0$; but, on the other hand, $y=0$, $p=0$ does not satisfy the differential equation, and is therefore not a solution at all, much less a singular solution.

Moreover, it is clear from all that precedes that, although it is true that when the differential equation

$$ay+bx^2+cxp+p^2=0$$

has a singular solution, its primitive is algebraic; yet, on the other hand, it is the exception, and not the rule, that it has a singular solution when its primitive is algebraic.

I have gone carefully into this matter, because the conclusion just arrived at shows that a proposition laid down by CAYLEY in connection with his well known Geometrical Theory of Singular Solutions of Differential Equations of the first order is erroneous, or at least very misleading. In his second paper on the subject (*Mess. Math.*, vol. vi. p. 23, 1877) the following passage occurs:—"Consider now a system of algebraic curves $U=0$, where U is, as regards (x, y), a rational integral function of the order m, and depends in any manner on an arbitrary parameter C, *I say that there is always a proper envelope*, which envelope is the singular solution of the differential equation obtained by the elimination of C from the equation $U=0$ and the derived equation in regard to (x, y). *It follows that the differential equation* $(L, M, N \between p, 1)^2=0$, *which has no singular solution, does not admit of an integral of the form in question*, $U=0$, *viz., an integral representing a system of algebraic curves.*"

CAYLEY, rests the above conclusion, which we have seen to be essentially erroneous, on a demonstration which amounts practically to this:—Consider an algebraic curve

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$U=0$ of order m, having singularities equivalent to δ double points and κ cusps. Of the intersections of $U=0$, with its consecutive $U+\delta_c U=0$, two will coincide with each double point and three with each cusp, leaving $m^2-2\delta-3\kappa$ other points of intersection. If n be the class of the curve $m^2-2\delta-3\kappa=m+n$, a number which cannot vanish; hence there is always an envelope, viz., the locus of these, $m^2-2\delta-3\kappa$ points.

The fallacy in this reasoning consists in assuming that the $m^2-2\delta-3\kappa$ points are necessarily spread out into a locus. It is, in fact, an inference that may be drawn from the above investigation that, *in general*, when a differential equation of the first order has an algebraic integral, this is not the case. So that only particular kinds of algebraic families can be integrals of equations of this description.

If we examine the particular case above discussed, for which $m=4$, $\delta=1$, $\kappa=2$* we see that $m^2-2\delta-3\kappa=8$. The eight points which ought, according to CAYLEY's theory, to form the envelope are concentrated, three at (0, 0) and the remaining five at $(0, \infty)$.

It is somewhat surprising that GLAISHER (*Mess. Math.*, xii. p. 2, 1883) seems to endorse the statement of CAYLEY just referred to, seeing that GLAISHER's examples (iii), (xiii), (xiv), (xv), (xx)† are instances to the contrary. It might be inferred from the somewhat guarded language used by FORSYTH (*Differential Equations*, § 30), and from his reference to CAYLEY, that he attaches some value to CAYLEY's result; but he has been kind enough to inform me that he is not to be understood as endorsing the proposition in dispute.

Trinodal Quartic Family which has no Envelope.—Inasmuch as the quartic family already discussed presents a peculiarity in respect that there is a condensed singularity; it may be well in the interest of the theory of envelopes to show that the degeneration of the envelope is not due to this cause. This we shall do by constructing a trinodal quartic whose singularities are all distinct and which has no proper envelope. The possibility of this may be seen *à priori* by considering that we can construct a quartic which has cusps at two given points, B and C, and given cuspidal tangents at these points, which passes through a given point A, and has a given tangent at A: this involves $5+5+2=12$ conditions. That there be a third node (position not specified) involves one more condition, leaving still one degree of freedom; so that we have a *family* of quartics fulfilling the given conditions. Now (as may be seen by discussing the intersection at the origin of the curves $y^2=cx^3$, $y^2=c^1x^3$‡) any two (and therefore

* The curves are unicursal quartics: by considering the intersection with the parabola $3y=\mu x(x-c)$ we find

$$x=4\mu c/(1+\mu)^2$$
$$3y=-4\mu^2(1-\mu)^2c^2(1+\mu)^4.$$

† There is an oversight in the interpretation of No. (xx).

‡ The two equations are equivalent to

$$y^2=cx^3,$$
$$(c-c^1)x^3=0;$$

that is to

$$x^3=0,$$
$$y^2=0,$$

which gave $x=0$, $y=0$ six times.

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any consecutive two) curves of this family intersect each other six times at B, six times at C, and twice at A. The remaining two points of intersection for two consecutive curves are infinitely near the double point. Hence the family has no proper envelope; but merely (so far as movable intersections are concerned) a node locus which counts twice.

Moreover, the quartic does not degenerate, as we shall show by working out its equation *in the special case where the point* A *is the intersection of the cuspidal tangent at* B *and* C.

Let us use triple coordinates and take A, B, C for triangle of reference: and let us write the equation to the quartic

$$a_0z^4+(ax+by)z^3+(cx^2+dxy+ey^2)z^2+(fx^3+gx^2y+hxy^2+ky^3)z+u_4=0 \quad . \quad (1).$$

Since $z=0$, which joins the cusps, must give $x^2=0$ and $y^2=0$, we see at once that $u_4\equiv x^2y^2$ (dropping the useless constant). Also, since $x=0$ and $y=0$ are cuspidal tangents, $x=0$ must give $z^3=0$, and $y=0$ must give $z^3=0$; hence we must have $e=0$, $k=0$, $c=0$, $f=0$.

The condition that the curve pass through (0, 0, 1) is obviously $a_0=0$; and, if we take $x+y=0$ to be the fixed tangent at (0, 0, 1), it is easily seen that we must have $a=b$. Our equation now reduces to

$$f(x,y,z)\equiv a(x+y)z^3+dxyz^2+(gx^2y+hxy^2)z+x^2y^2=0 \quad . \quad . \quad . \quad (2).$$

We have

$$\begin{aligned}
f_x &\equiv az^3+dyz^2+(2gxy+hy^2)z+2xy^2;\\
f_y &\equiv az^3+dxz^2+(2hxy+gx^2)z+2x^2y;\\
f_z &\equiv 3a(x+y)z^2+\qquad 2dxyz+(gx+hy)xy;\\
f_{zz} &\equiv 2gyz+2y^2;\\
f_{yy} &\equiv 2hxz+2x^2;\\
f_{zx} &\equiv 6a(x+y)z+2dxy;\\
f_{yz} &\equiv 3az^2+2dxz+2hxy+gx^2;\\
f_{zx} &\equiv 3az^2+2dyz+2gxy+hy^2;\\
f_{xy} &\equiv dz^2+2(gx+hy)z+4xy.
\end{aligned}$$

It will be seen that the conditions for double points at (0, 1, 0) and (1, 0, 0) are satisfied. In order that these points may be cusps, it is farther necessary that $\Sigma f_{zz}\xi^2+2\Sigma f_{yz}\eta\zeta$ shall be a complete square in ξ, η, ζ at the two points. Now this function reduces to $2\xi^2+2h\zeta\xi$ and $2\eta^2+2g\eta\zeta$ at (0, 10) and (1, 0, 0) respectively; hence we must have $g=0$, $h=0$.

Our equation is now reduced to

$$a(x+y)z^3+dxyz^2+x^2y^2=0 \quad . \quad . \quad . \quad . \quad . \quad (3);$$

and it remains only to find the condition that the quartic have a third node. The conditions for a node are

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$$az^3+dyz^2+2xy^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4),$$
$$az^3+dxz^2+2x^2y=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5),$$
$$3a(x+y)z^2+2dxyz=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

Setting aside alternatives which lead to $z=0$, $x=0$ and $z=0$, $y=0$ (double points already existing), we find that (4) and (5) are equivalent to

$$az^3+dxz^2+2dx^3=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7),$$
$$x-y=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

The admissible solution or solutions of (7) and (8) must also satisfy (6). Hence, setting aside $x=0$, as before, we must have

$$3az+dx=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

Substituting the value of z/x given by (9) in (7) we get for the necessary and sufficient condition required

$$d^3+27a^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10),$$

which will be satisfied if we take $a=c^3$, $d=-3c^2$. The coordinates of the third node are then $x=y=c$, $z=1$.

For these values

$$\Sigma f_{\xi\xi}\xi^2+2\Sigma f_{\xi\eta}\eta\zeta$$

reduces to

$$2c^2\xi^2+2c^2\eta^2+6c^4\zeta^2-6c^3\eta\zeta-6c^3\zeta\xi+2c^2\xi\eta\equiv 2c^2\{\xi-\omega\eta+(\omega-1)c\zeta\}\{\xi-\omega^2\eta+(\omega^2-1)c\zeta\},$$

where ω is an imaginary cube root of unity. *The node is, therefore, an acnode.*

The quartic family,

$$c^3(x+y)z^3-3c^2xyz^2+x^2y^2=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (11),$$

therefore, has fixed cusps at (1, 0, 0) *and* (0, 1, 0), *the cuspidal tangents being* $y=0$ *and* $x=0$, *a movable acnode* $x=y=c$, $z=1$; *each curve passes through the intersection of its cuspidal tangents and has a fixed tangent at that point.*

As a verification of the result already predicted, we may calculate the coordinates of the intersections of two consecutive curves of the family. These are given by

$$\left.\begin{aligned} c^3(x+y)z^3-3c^2xyz^2+x^2y^2&=0,\\ c(x+y)z^3-2xyz^2&=0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12).$$

Now (12) is equivalent to the system

$$z^2=0, x^2y^2=0,$$

which gives (1, 0, 0) and (0, 1, 0) each four times, together with

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$$\left.\begin{aligned} c^3(x+y)z^3-3c^2xyz^2+x^2y^2=0, \\ c(x+y)z-2xy=0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (13).$$

This last is equivalent to

$$\left.\begin{aligned} c^3(x+y)z^3-3c^2xyz^2+\tfrac{1}{4}c^2(x+y)^2z^2=0, \\ c(x+y)z-2xy \quad =0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (14).$$

(14) is equivalent to $z^2=0$, $xy=0$, which gives (1, 0, 0) and 0, 1, 0) each twice, together with

$$\left.\begin{aligned} c(x+y)z-3xy+\tfrac{1}{4}(x+y)^2=0, \\ c(x+y)z-2xy=0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15);$$

that is,

$$\left.\begin{aligned} (x-y)^2=0, \\ c(x+y)z-2xy=0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16);$$

that is,

$$\left.\begin{aligned} (x-y)^2=0, \\ x(cz-x)=0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17).$$

Now (17) gives (0, 0, 1) and (c, c, 1) each twice. Hence the intersections are (1, 0, 0) six times, (0, 1, 0) six times, (0, 0, 1) twice, and (c, c, 1) twice. There is, therefore, a node locus, but no envelope.

As a farther verification we may calculate the c-discriminant of (11). The result is

$$27z^6x^4y^4(x-y)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18).$$

The factor $(x-y)^2$ corresponds to the node locus, the factors x^4, y^4, z^6 to the sides of the triangle of reference, which are parts of degenerate transition quartics of the system.

By means of a conic passing through A, B, C and the acnode, it is easy to obtain rational expressions for the ratios of the coordinates in terms of a parameter θ. The result is

$$\left.\begin{aligned} x/y=-2c\theta(\theta+c)/(\theta-c)^2, \\ y/z=-2c\theta(\theta-c)/(\theta+c)^2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19),$$

by means of which this interesting family of curves may readily be traced.

It would be easy to construct a large number of examples of the degeneration of the envelope of an algebraic family. It will be sufficient to add a pair of examples of cubics which have this property.

The cubic

$$z^3+c^3xy(x+y)-3c^2xyz=0$$

has real inflexions at (0, 1, 0) *and* (1, 0, 0), *the inflexional tangents being* $x=0$, $y=0$ *respectively, passes through the fixed point* (1, -1, 0) *and has an acnode at* (1, 1, c).

Any two consecutive curves of the family intersect thrice at (0, 1, 0), thrice at

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(1, 0, 0), once at (1, -1, 0), and twice at (1, 1, c). The family has no envelope; but merely the node locus $x-y=0$.

The c-discriminant locus is

$$27x^6x^2y^2(x-y)^2=0.$$

The cubic

$$9c^2x^2-4cx-(12y^3+12y^2+4y)=0,$$

or

$$x-\frac{2}{9c}\{1\pm(3y+1)^{\frac{3}{2}}\}$$

has a fixed inflexion and inflexional tangent at ∞ *and passes through the fixed points*

$$\overset{\text{A.}}{(0,0)},\quad \overset{\text{B.}}{\left(0,\frac{-3+i\sqrt{3}}{6}\right)},\quad \overset{\text{C.}}{\left(0,\frac{-3-i\sqrt{3}}{6}\right)}.$$

It has a cusp at (2/9c, $-1/3$). *The intersections of two consecutive curves are* (∞, 0) *thrice,* A, B, C *and the cusp thrice. There is therefore merely a cusp locus* (fig. 4).

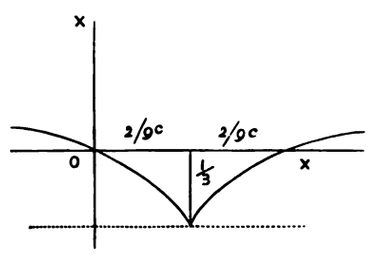

FIG. 5.

The c-discriminant locus is

$$16x^2(3y+1)^3=0.$$

$x^2=0$ corresponds to a degenerate curve of the family for which $c=\infty$.

The simplest case of all in which the envelope degenerates into a point-discrete is the linear pencil

$$cu+v=0,$$

where u and v are integral functions of x and y. In this case each curve intersects every other consecutive or non-consecutive in the points of intersection of $u=0$, $v=0$; and the corresponding differential equation of the first order and first degree has, as is well known, no envelope singular solution. The proper consideration of this case alone is sufficient to show that no theory of envelopes which takes no account of the manner in which the arbitrary constant is involved can be of much value for the theory of differential equations.

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APPENDIX.

TRANSACTIONS

OF THE

ROYAL SOCIETY OF EDINBURGH.

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COUNCIL,

ALPHABETICAL LIST OF ORDINARY FELLOWS,

AND LIST OF HONORARY FELLOWS,

At January 1897.

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THE COUNCIL

OF

THE ROYAL SOCIETY OF EDINBURGH

JANUARY 1897.

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ALPHABETICAL LIST

OF

THE ORDINARY FELLOWS OF THE SOCIETY.

CORRECTED TO JANUARY 1897.

N.B.—*Those marked * are Annual Contributors.*

B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.
K. ,, ,, ,, Keith Medal.
N. ,, ,, ,, Neill Medal.
V. J. ,, ,, ,, the Gunning Victoria Jubilee Prize.
C. ,, ,, ,, contributed one or more Communications to the Society's TRANSACTIONS or PROCEEDINGS.

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1892		* Coates, Henry, Pitcullen House, Perth
1887		* Cockburn, John, F.R.A.S., Glencorse House, Milton Bridge, Midlothian
1888	C.	Collie, John Norman, Ph.D., F.R.S., F.C.S., Professor of Chemistry to the Pharmaceutical Society of Great Britain, 17 Bloomsbury Square, London 110
1886		Connan, Daniel M., M.A., Education Department, Cape of Good Hope
1872		Constable, Archibald, LL.D., 11 Thistle Street
1894		Cook, John, M.A., Principal of the Central College, Bangalore, India
1891		* Cooper, Charles A., 15 Charlotte Square, Edinburgh
1890	C.	* Copeland, Ralph, Ph.D., Astronomer-Royal for Scotland, and Professor of Practical Astronomy in the University of Edinburgh (VICE-PRESIDENT), Royal Observatory, Blackford Hill, Edinburgh 115
1879		* Cox, Robert, of Gorgie, M.A., M.P. for the Southern Division of Edinburgh, 34 Drumsheugh Gardens
1870		Crichton-Browne, Sir Jas., M.D., LL.D., F.R.S., Lord Chancellor's Visitor and Vice-President of the Royal Institution of Great Britain, 61 Carlisle Place Mansions, Victoria Street, and Royal Courts of Justice, Strand, London
1875		* Craig, William, M.D., F.R.C.S.E., Lecturer on Materia Medica to the College of Surgeons, 71 Bruntsfield Place

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Date of Election.		
1887		* Crawford, William Caldwell, Lockharton Gardens, Slateford, Edinburgh
1886		* Croom, John Halliday, M.D., F.R.C.P.E., 25 Charlotte Square 120
1878		* Cunningham, Daniel John, M.D., D.C.L., F.R.S., F.Z.S., Professor of Anatomy in Trinity College, Dublin, 43 Fitzwilliam Place, Dublin
1871		Cunynghame, R. J. Blair, M.D., Vice-President of the Royal College of Surgeons, 18 Rothesay Place
1885		* Daniell, Alfred, M.A., LL.B., D.Sc., Advocate, 3 Great King Street
1884		Davy, Richard, F.R.C.S., Surgeon to the Westminster Hospital, Burstone House, Bow, North Devon
1894		* Denny, Archibald, Braehead, Dumbarton 125
1895		* Deuchar, David, F.I.A., F.F.A., Actuary, 12 Hope Terrace
1869	C.	Dewar, James, M.A., LL.D., F.R.S., Jacksonian Professor of Natural and Experimental Philosophy in the University of Cambridge, and Fullerian Professor of Chemistry at the Royal Institution of Great Britain, London
1884		* Dickson, Charles Scott, Advocate, Solicitor-General for Scotland, 4 Heriot Row
1888	C.	* Dickson, H. N., 2 St Margaret's Road, Oxford
1876	C.	* Dickson, J. D. Hamilton, M.A., Fellow and Tutor, St Peter's College, Cambridge 130
1885	C.	Dixon, James Main, M.A., Professor of English Literature in the Washington University of St Louis, United States
1881	C.	* Dobbin, Leonard, Ph.D., Assistant to the Professor of Chemistry in the University of Edinburgh, 7 Cobden Road
1867	C.	Donaldson, J., M.A., LL.D., Principal of the University of St Andrews, St Andrews
1896		* Donaldson, William, M.A., Viewpark House
1882	C.	* Dott, D. B., Memb. Pharm. Soc., 29 Spring Gardens, E. 135
1892		Doyle, Patrick, C.E., M.R.I.A., F.G.S., Editor of *Indian Engineering*, Calcutta
1866		Douglas, David, 22 Drummond Place
1880	C.	* Drummond, Henry, F.G.S., Professor of Natural History in the Free Church College, 2 Park Circus, Glasgow
1876		* Duncan, James, 9 Mincing Lane, London
1889		* Duncan, James Dalrymple, F.S.A. Lond. and Scot., Meiklewood, Stirling 140
1870		Duncan, John, M.A., M.D., LL.D., F.R.C.S.E., 8 Ainslie Place
1878		* Duncanson, J. J. Kirk, M.D., F.R.C.P.E., 22 Drumsheugh Gardens
1859	C.	Duns, Rev. Professor, D.D., New College, Edinburgh, 5 Greenhill Place
1892		Dunstan, M. J. R., B.A., F.C.S., Director of Technical Education in Agriculture, Newcastle Circus, The Park, Nottingham
1888		* Durham, James, F.G.S., Wingate Place, Newport, Fife 145
1893		Edington, Alexander, M.B., C.M., Colonial Bacteriologist, Graham's Town, South Africa
1869		Elder, George, Knock Castle, Wemyss Bay, Greenock
1885		Elgar, Francis, Memb. Inst. C.E., LL.D., F.R.S., 18 York Terrace, Regent's Park, London
1875		Elliot, Daniel G., Curator of the Department of Zoology, Field Columbian Museum, Chicago, U.S.
1855		Etheridge, Robert, F.R.S., Assistant-Keeper of the Geological Department at the British Museum of Natural History, 14 Carlyle Square, Chelsea, London 150
1884		* Evans, William, F.F.A., 18A Morningside Park
1863	C.	Everett, J. D., M.A., D.C.L., F.R.S., Professor of Natural Philosophy, Queen's College, Belfast
1879	C.	* Ewart, James Cossar, M.D., F.R.C.S.E., F.R.S., F.L.S., Professor of Natural History, University of Edinburgh

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Date of Election.			
1878	C.	* Ewing, James Alfred, B.Sc., Memb. Inst. C.E., F.R.S., Professor of Mechanism and Applied Mechanics in the University of Cambridge, Langdale Lodge, Cambridge	
1875		Fairley, Thomas, Lecturer on Chemistry, 8 Newton Grove, Leeds	155
1888	C.	* Fawsitt, Charles A., 9 Foremount Terrace, Dowanhill, Glasgow	
1859		Fayrer, Sir Joseph, Bart., K.C.S.I., M.D., LL.D., F.R.C.P.L., F.R.C.S. L. and E., F.R.S., Honorary Physician to the Queen, 16 Devonshire Street, Portland Place, London, W.	
1883	C.	* Felkin, Robert W., M.D., F.R.G.S., Fellow of the Anthropological Society of Berlin, 8 Alva Street	
1888		* Ferguson, John, M.A., LL.D., Professor of Chemistry in the University of Glasgow	
1868	C.	Ferguson, Robert M., LL.D., Ph.D., 3 Learmonth Terrace	160
1886		Field, C. Leopold, F.C.S., Upper Marsh, Lambeth, London	
1852		Fleming, Andrew, M.D., Deputy Surgeon-General, 8 Napier Road	
1876		* Fleming, J. S., 16 Grosvenor Crescent	
1880		* Flint, Robert, D.D., Corresponding Member of the Institute of France, Corresponding Member of the Royal Academy of Sciences of Palermo, Professor of Divinity in the University of Edinburgh (VICE-PRESIDENT), Johnstone Lodge, 54 Craigmillar Park	
1872	C.	Forbes, Professor George, M.A., Memb. Inst. C.E., M.S.T.E. and E., F.R.S., F.R.A.S., 34 Great George Street, Westminster	165
1892		* Ford, John Simpson, F.C.S., 11 Abbotsford Park	
1859		Forlong, Major-Gen. J. G., F.R.G.S., R.A.S., Assoc. C.E., &c., 11 Douglas Crescent	
1828		Foster, John, Liverpool	
1887		Fowler, Sir John, Bart., K.C.M.G., Memb. Inst. C.E., LL.D., Thornwood Lodge, Kensington, London	
1858		Fraser, A. Campbell, M.A., LL.D., D.C.L., Emeritus Professor of Logic and Metaphysics in the University of Edinburgh, Gorton House, Hawthornden	170
1896		* Fraser, John, M.B., F.R.C.P.E., one of H.M. Commissioners in Lunacy for Scotland, 19 Strathearn Road	
1892		* Fraser, Patrick Neill, Rockville, Murrayfield	
1867	K. B. C.	Fraser, Thomas R., M.D., F.R.C.P.E., LL.D., F.R.S., Professor of Materia Medica in the University of Edinburgh, 13 Drumsheugh Gardens	
1891		* Fullarton, J. H., M.A., D.Sc., Zoologist to the Fishery Board for Scotland, 101 George Street	
1891		* Fulton, T. Wemyss, M.B., Secretary for Scientific Investigations to the Scottish Fishery Board, 30 Dalhousie Terrace	175
1892		* Fyfe, Peter, Chief Sanitary Inspector, Glasgow	
1888		* Galt, Alexander, B.Sc., F.C.S., Physical Laboratory, The University, Glasgow	
1894		Gatty, Charles Henry, M.A., LL.D., F.L.S., Felbridge Place, East Grinstead	
1867		Gayner, Charles, M.D., Oxford	
1889		* Geddes, George H., Mining Engineer, 8 Douglas Crescent	180
1880	C.	* Geddes, Patrick, Professor of Botany in University College, Dundee, and Lecturer on Zoology, Ramsay Garden, University Hall, Edinburgh	
1861	B. C.	Geikie, Sir Archibald, LL.D., D.Sc. Dub., F.R.S., F.G.S., Corresponding Member of the Institute of France, Corresponding Member of the Royal Academy of Berlin, Director of the Geological Surveys of Great Britain, and Head of the Geological Museum, 28 Jermyn Street, London	
1871	B. C.	Geikie, James, LL.D., D.C.L., F.R.S., F.G.S., Professor of Geology in the University of Edinburgh (VICE-PRESIDENT), 31 Merchiston Avenue	
1881		* Gibson, George Alexander, D.Sc., M.D., F.R.C.P.E., 17 Alva Street	

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Date of Election.		
1890		* Gibson, George A., M.A., Professor of Mathematics in the Glasgow and West of Scotland Technical College, 183 Renfrew Street, Glasgow 185
1877	C.	* Gibson, John, Ph.D., Professor of Chemistry in the Heriot-Watt College, 20 George Square
1892		Gifford, H. J., Walsingham House, Piccadilly, London, W.
1887		* Gilmour, William, 9 Inverleith Row
1880		* Gilruth, George Ritchie, Surgeon, 48 Northumberland Street
1850		Gosset, Major-General W. D., R.E., 70 Edith Road, West Kensington, London 190
1880		* Graham, James, LL.D., 198 West George Street, Glasgow
1891		* Graham, Richard D., 11 Strathearn Road
1883		* Gray, Andrew, M.A., LL.D., F.R.S., Professor of Physics in University College, Bangor, North Wales
1880	C.	Gray, Thomas, B.Sc., Prof. of Physics, Rose Polytechnic Institute, Terre Haute, Indiana, U.S.
1886		* Greenfield, W. S., M.D., F.R.C.P.E., Professor of General Pathology in the University of Edinburgh, 7 Heriot Row 195
1886	C	* Griffiths, Arthur Bower, Ph.D., Lecturer at the National Dental Hospital and College, London, 12 Knowle Road, Brixton, London
1883		Gunning, His Excellency Robert Halliday, Grand Dignitary of the Order of the Rose of Brazil, M.A., M.D., LL.D., 12 Addison Crescent, Kensington
1888	C.	Guppy, Henry Brougham, M.B., Craven Villa, Matlock Dale, Derbyshire
1867		Hallen, James H. B., C.I.E., F.R.C.S.E., Veterinary Lieut.-Colonel in H.M. Indian Army, Retired, Pebworth Fields, under Stratford-on-Avon
1881	C.	* Hamilton, D. J., M.B., F.R.C.S.E., Professor of Pathological Anatomy in the University of Aberdeen, 4 Forest Road, Aberdeen 200
1876	C.	* Hannay, J. Ballantyne, Cove Castle, Loch Long
1896		* Harris, David, Fellow of the Statistical Society, West Grange, Grange Loan, Edinburgh
1896	C.	* Harris, David Fraser, B.Sc., M.B., C.M., Assistant to the Professor of Physiology in the University of Glasgow, West Grange, Grange Loan, Edinburgh
1888		* Hart, D. Berry, M.D., F.R.C.P.E., 29 Charlotte Square
1869		Hartley, Sir Charles A., K.C.M.G., Memb. Inst. C.E., 26 Pall Mall, London 205
1877	C.	Hartley, W. N., F.R.S., Prof. of Chemistry, Royal College of Science for Ireland, Dublin
1881		* Harvie-Brown, J. A., of Quarter, Dunipace House, Larbert, Stirlingshire
1880	C.	* Haycraft, J. Berry, M.D., D.Sc., Professor of Physiology in the University College of South Wales and Monmouthshire, Cardiff
1892		* Heath, Thomas, B.A., Assistant Astronomer, Royal Observatory, Edinburgh
1862		Hector, Sir J., K.C.M.G., M.D., F.R.S., Director of the Geological Survey, Wellington, New Zealand 210
1876	K. C.	* Heddle, M. Forster, M.D., Emeritus Prof. of Chemistry in the University of St Andrews
1893		Hehir, Patrick, M.D., F.R.C.S.E., M.R.C.S.L., L.R.C.P.E., Surgeon-Captain, Indian Medical Service, Principal Medical Officer, H.H. the Nizam's Army, Hyderabad, Deccan, India
1890	C.	Helme, T. A., M.D., 258 Oxford Road, Manchester
1884		* Henderson, John, Meadowside Works, Partick, Glasgow
1890	C.	* Hepburn, David, M.D., Lecturer on Regional Anatomy in the University of Edin. 215
1896	C.	* Herbertson, Andrew J., Lecturer on Commercial Geography in the Heriot-Watt College, The Loan, Colinton
1881	N. C.	* Herdman, W. A., D.Sc., F.R.S., Prof. of Natural History in University College, Liverpool
1871		Higgins, Charles Hayes, M.D., M.R.C.P.L., F.R.C.S., Alfred House, Birkenhead

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Date of Election.		
1894		Hill, Alfred, M.D., M.R.C.S., F.I.C., Medical Officer of Health, The Council House, Birmingham
1859		Hills, John, Major-General, C.B., Bombay Engineers, United Service Club, London, and Love's Grove, Aberystwith, Wales 220
1879		Hislop, John, LL.D., formerly Secretary to the Department of Education, Forth Street, Dunedin, New Zealand
1885		Hodgkinson, W. R., Ph.D., F.I.C., F.C.S., Professor of Chemistry and Physics at the Royal Military Academy and Royal Artillery College, Woolwich, 8 Park Villas, Blackheath, Kent
1881	N. C.	* Horne, John, F.G.S., Geological Survey of Scotland, Sheriff-Court Buildings, Edinburgh
1896		Horne, J. Fletcher, M.D., F.R.C.S.E., The Poplars, Barnsley
1893		Howden, Robert, M.B., C.M., Lecturer on Anatomy, University of Durham College of Medicine, Newcastle-on-Tyne 225
1883	C.	* Hoyle, William Evans, M.A., M.R.C.S., 25 Brunswick Road, Withington, Manchester
1886		Hunt, Rev. H. G. Bonavia, Mus. D. Dub., Mus. B. Oxon., F.L.S., La Belle Sauvage, London
1872		Hunter, Colonel Charles, of Pläs Cöch, Llanfairpwll, Anglesea, and Junior United Service Club, London
1887	C.	* Hunter, James, F.R.C.S.E., F.R.A.S., Rosetta, Liberton, Midlothian
1887	C.	* Hunter, William, M.D., M.R.C.P. L. and E., M.R.C.S., 54 Harley Street, London 230
1882		* Inglis, J. W., Memb. Inst. C.E., Kenwood, Liberton, Midlothian
1886	N. C.	* Irvine, Robert, Royston, Granton, Edinburgh
1875		Jack, William, M.A., LL.D., Professor of Mathematics in the University of Glasgow
1894		Jackson, Sir John, 10 Holland Park, London
1889	C.	* James, Alexander, M.D., F.R.C.P.E., 44 Melville Street 235
1882		* Jamieson, A., Memb. Inst. C.E., Professor of Engineering in The Glasgow and West of Scotland Technical College, Glasgow
1860	C.	Jamieson, George Auldjo, Actuary, 24 St Andrew Square
1880		Japp, A. H., LL.D., The Limes, Elmstead, near Colchester
1869		Johnston, John Wilson, M.D., Surgeon Lieut.-Colonel, Benmore, 30 Bidston Road, Oxton, Cheshire
1895		* Johnston, Surgeon-Major Henry Halcro, D.Sc., M.D., Orphir House, Kirkwall, Orkney 240
1867		Johnston, T. B., F.R.G.S., Geographer to the Queen, 9 Claremont Crescent
1874		Jones, Francis, Lecturer on Chemistry, Beaufort House, Alexandra Park, Manchester
1888		Jones, John Alfred, Memb. Inst. C.E., Vice-President, and Engineer, City of Madras, Peter's Road, Madras
1896		* Jong, E. F. de, L.R.C.P.E., L.R.C.S.E., F.R.C.V.S., Jock's Lodge, Edinburgh
1847	K. C. V. J.	Kelvin, The Right Hon. Lord, G.C.V., LL.D., D.C.L., F.R.S. (PRESIDENT), Grand Officer of the Legion of Honour of France, Member of the Prussian Order *Pour le Mérite*, Foreign Associate of the Institute of France, and Regius Professor of Natural Philosophy in the University of Glasgow 245
1892		* Kerr, Rev. John, M.A., Manse, Dirleton
1891		Kerr, Joshua Law, M.D., Croft House, Crawshawbooth, Manchester
1886	N. C.	* Kidston, Robert, F.G.S., 24 Victoria Place, Stirling
1877		* King, Sir James, of Campsie, Bart., LL.D., 115 Wellington Street, Glasgow
1880		* King, W. F., Lonend, Russell Place, Trinity 250

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Date of Election.		
1886		* Kingsburgh, The Right Hon. Lord, C.B., Q.C., LL.D., F.R.S., M.S.T.E. and E., Lord Justice-Clerk, and Lord President of the Second Division of the Court of Session, 15 Abercromby Place
1883		* Kinnear, The Right Hon. Lord, one of the Senators of the College of Justice, 2 Moray Place
1878		* Kintore, The Right Hon. the Earl of, M.A. Cantab., Keith Hall, Inglismaldie Castle, Laurencekirk
1880	K. C.	* Knott, C. G., D.Sc., Lecturer on Applied Mathematics in the University of Edinburgh (late Prof. of Physics, Imperial University, Japan), 42 Upper Gray Street, Edinburgh
1896		* Kuenen, J. P., Ph.D. (Leiden), Prof. of Natural Philosophy in University College, Dundee 255
1886		* Laing, Rev. George P., 17 Buckingham Terrace
1878	C.	* Lang, P. R. Scott, M.A., B.Sc., Professor of Mathematics in the University of St Andrews
1885	C.	* Laurie, A. P., B.A., B.Sc., Woodside, Baldwin's Hill, Loughton
1894	C.	* Laurie, Malcolm, B.A., D.Sc., F.L.S., Professor of Zoology, St Mungo's College, Glasgow
1870		Laurie, Simon S., M.A., LL.D., Professor of Education in the University of Edinburgh, 22 George Square 260
1872		Lee, Alexander H., C.E., 58 Manor Place
1863		Leslie, Hon. G. Waldegrave, Leslie House, Leslie
1874	K. C.	* Letts, E. A., Ph.D., F.I.C., F.C.S., Professor of Chemistry, Queen's College, Belfast
1889		* Lindsay, Rev. James, B.D., B.Sc., F.G.S., Corresponding Member of the Royal Academy of Sciences, Letters and Arts, of Padua, Minister of St Andrew's Parish, Springhill Terrace, Kilmarnock
1870	B. C.	Lister, The Right Hon. Lord, M.D., F.R.C.S.L., F.R.C.S.E., LL.D., D.C.L., P.R.S., Foreign Associate of the Institute of France, Professor of Clinical Surgery, King's College, Surgeon Extraordinary to the Queen, 12 Park Crescent, Portland Place, London 265
1895		* Lothian, The Most Hon. The Marquis of, K.T., LL.D., Newbattle Abbey, Dalkeith
1884		* Low, George M., Actuary, 15 Chester Street
1888		* Lowe, D. F., M.A., Headmaster of Heriot's Hospital School, Lauriston
1849		Lowe, W. H., M.D., F.R.C.P.E., Woodcote, Inner Park, Wimbledon
1886		Lyster, George Fosbery, Memb. Inst. C.E., Gisburn House, Liverpool 270
1894		* Mabbott, Walter John, M.A., Rector of County High School, Duns, Berwickshire
1855		Macadam, Stevenson, Ph.D., Lecturer on Chemistry, Surgeons' Hall, Edinburgh, 11 East Brighton Crescent, Portobello
1888		* Macadam, W. Ivison, F.I.C., F.C.S., Lecturer on Chemistry, Slioch, Lady Road, Newington, Edinburgh
1887		M'Aldowie, Alexander M., M.D., 6 Brook Street, Stoke-on-Trent
1891		Macallan, John, F.I.C., 3 Charlemont Terrace, Clontarf, Dublin 275
1888	C.	M'Arthur, John, F.C.S., 196 Trinity Road, Wandsworth Common, London
1885		* M'Bride, Charles, M.D., Wigtown
1883		* M'Bride, P., M.D., F.R.C.P.E., 16 Chester Street
1867		M'Candlish, John M., W.S., 27 Drumsheugh Gardens
1894		* Macdonald, James, Secretary of the Highland and Agricultural Society of Scotland, 9 Lauriston Gardens 280
1886		* Macdonald, William J., M.A., Comiston Drive
1888	C.	* M'Fadyean, John, M.B., B.Sc., Professor of Pathology and Dean of the Royal Veterinary College, Camden Town, London
1878	C.	Macfarlane, Alexander, M.A., D.Sc., LL.D., Lecturer in Physics in Lehigh University, Pennsylvania

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Date of Election.			
1885	C.	* Macfarlane, J. M., D.Sc., Professor of Biology in the University of Pennsylvania, Lansdowne, Delaware Co., Pennsylvania	
1878		* M'Gowan, George, F.I.C., Ph.D., 1 Mount Avenue, Ealing, Middlesex	285
1886		* MacGregor, Rev. James, D.D., 3 Eton Terrace	
1880	C.	MacGregor, J. G., M.A., D.Sc., Professor of Physics in Dalhousie College, Halifax, Nova Scotia	
1869	N.C.	M'Intosh, William Carmichael, M.D., LL.D., F.R.S., F.L.S., Professor of Natural History in the University of St Andrews, 2 Abbotsford Crescent, St Andrews	
1895	C.	* Macintyre, John, M.D., 179 Bath Street, Glasgow	
1882		* Mackay, John Sturgeon, M.A., LL.D., Mathematical Master in the Edinburgh Academy, 69 Northumberland Street	290
1873	B. C.	* M'Kendrick, John G., M.D., F.R.C.P.E., LL.D., F.R.S. (Vice-President), Professor of Physiology in the University of Glasgow	
1840		Mackenzie, John, New Club, Princes Street	
1894		* Mackenzie, Robert, M.D., 1 Bruntsfield Terrace	
1843	C.	Maclagan, Sir Douglas, M.D., F.R.C.P.E., LL.D., (Honorary Vice-President), Emeritus Professor of Medical Jurisprudence in the University of Edinburgh, 28 Heriot Row	
1894		* Maclagan, Philip R. D., F.F.A. (Treasurer), St Catherine's, Liberton	295
1869		Maclagan, R. C., M.D., F.R.C.P.E., 5 Coates Crescent	
1864		M'Lagan, Peter, of Pumpherston	
1869	C.	M'Laren, The Hon. Lord, LL.D. Edin. and Glasg., F.R.A.S., one of the Senators of the College of Justice (Vice-President), 46 Moray Place	
1888		* Maclean, Magnus, M.A., D.Sc., Lecturer on Physics in the University of Glasgow, 8 St Alban's Terrace, Dowanhill, Glasgow	
1876		* Macleod, Rev. Norman, D.D., Westwood, Inverness	300
1883		* Macleod, W. Bowman, L.D.S., 16 George Square	
1896		* M'Lintock, James, M.D., B.Sc., Member of the Local Government Board of Scotland, 5 Atholl Crescent	
1872		Macmillan, Rev. Hugh, D.D., LL.D., 70 Union Street, Greenock	
1876		* Macmillan, John, M.A., D.Sc., M.B., C.M., F.R.C.P.E., 27 Warrender Park Road	
1893		* M'Murtrie, The Rev. John, M.A., D.D., 5 Inverleith Place	305
1884		* Macpherson, Rev. J. Gordon, M.A., D.Sc., Ruthven Manse, Meigle	
1888		Mactear, James, F.C.S., 2 Victoria Mansions, Hyde Park, London	
1890		* M'Vail, John C., M.D., 2 Strathallan Terrace, Dowanhill, Glasgow	
1880	C.	Marsden, R. Sydney, M.B., C.M., D.Sc., F.I.C., F.C.S., 64 Park Road, South, and Town Hall, Birkenhead	
1882	C.	Marshall, D. H., M.A., Professor of Physics in Queen's University and College, Kingston, Ontario, Canada	310
1869		Marshall, Henry, M.D., Clifton, Bristol	
1888	C.	* Marshall, Hugh, D.Sc., Assistant to the Professor of Chemistry in the University of Edinburgh, 131 Warrender Park Road	
1892		* Martin, Francis John, W.S., 9 Glencairn Crescent	
1864		Marwick, Sir James David, LL.D., Town-Clerk, Glasgow	
1866		Masson, David, LL.D., Litt.D. Dub., Professor of Rhetoric and English Literature in the Univ. of Edinburgh, Her Majesty's Historiographer for Scotland, 110 Princes Street	315
1885	C.	* Masson, Orme, D.Sc., Professor of Chemistry in the University of Melbourne	
1890		* Matheson, The Rev. George M.A., B.D., D.D., Minister of St Bernard's, Edinburgh, 19 St Bernard's Crescent	

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Date of Election.		
1888		* Methven, C. W., Memb. Inst. C.E., Engineer-in-Chief to the Natal Harbour Board, Equitable Buildings, Durban
1885	B. C.	* Mill, Hugh Robert, D.Sc., Librarian, Royal Geographical Society, 1 Saville Row, London
1833		Milne, Admiral Sir Alexander, Bart., G.C.B., Inveresk 320
1886		* Milne, William, M.A., B.Sc., 57 Springbank Terrace, Aberdeen
1866		Mitchell, Sir Arthur, K.C.B., M.A., M.D., LL.D., 34 Drummond Place
1889	C.	* Mitchell, A. Crichton, D.Sc., Professor of Pure and Applied Mathematics, and Principal of the Maharajah's College, Trivandrum, Travancore, India
1865		Moir, John J. A., M.D., F.R.C.P.E., 52 Castle Street
1871		Moncrieff, Rev. Canon William Scott, of Fossaway, Easington Rectory, Castle Eden, County Durham 325
1890	C.	Mond, R. L., M.A. Cantab., F.C.S., 20 Avenue Road, Regent's Park, London
1868		Montgomery, Very Rev. Dean, M.A., D.D., 17 Atholl Crescent
1887		Moos, N. A. F., L.C.E., B.Sc., Assistant Prof. of Engineering, College of Science, Bombay
1896		* Morgan, Alexander, M.A., B.Sc., 6 Cluny Terrace
1892		Morrison, J. T., M.A., B.Sc., Professor of Physics and Chemistry, Victoria College, Stellenbosch, Cape Colony 330
1892	C.	* Mossman, Robert C., 10 Blacket Place
1874	K. C.	* Muir, Thomas, M.A., LL.D., Superintendent-General of Education for Cape Colony, Education Office, and The Hall, Mowbray, Cape Town
1888		* Muirhead, George, Mains of Haddo, Aberdeen
1887		Mukhopâdhyay, Âsûtosh, M.A., LL.D., F.R.A.S., M.R.I.A., Professor of Mathematics at the Indian Association for the Cultivation of Science, 77 Russa Road North, Bhowanipore, Calcutta
1870		Munn, David, M.A., 1 Albyn Place 335
1894		* Munro, J. M. M., M.I.E.E., 136 Bothwell Street, Glasgow
1889		* Munro, Rev. Robert, M.A., B.D., F.S.A. Scot., Free Church Manse, Old Kilpatrick
1891	C.	* Munro, Robert, M.A., M.D., Hon. Memb. R.I.A., Hon. Memb. Royal Soc. of Antiquaries of Ireland, Secretary of the Society of Antiquaries of Scotland, 48 Manor Place
1896		* Murray, Alfred A., M.A., LL.B., 20 Warriston Crescent
1892	C.	* Murray, George Robert Milne, F.R.S., F.L.S., Keeper of the Botanical Department, British Museum, (Natural Hist.), Cromwell Road, London 340
1857		Murray, John Ivor, M.D., F.R.C.S.E., M.R.C.P.E., 24 Huntriss Row, Scarborough
1877	B. N. C.	* Murray, John, LL.D., Ph.D., D.Sc., F.R.S. (Secretary), (Society's Representative on George Heriot's Trust), Director of the Challenger Expedition Publications. Office, 45 Frederick St. House, Challenger Lodge, Wardie, Edin., and United Service Club
1888		* Murray, R. Milne, M.A., M.B., F.R.C.P.E., 11 Chester Street
1887		Muter, John, M.A., F.C.S., South London Central Public Laboratory, 325 Kennington Road, London
1888		Napier, A. D. Leith, M.D., C.M., M.R.C.P.L., 67 Grosvenor St., Grosvenor Sq., London 345
1895		* Napier, James, M.A., Drums, Old Kilpatrick
1877		* Napier, John, C. Audley Mansions, Grosvenor Square, London
1887		* Nasmyth, T. Goodall, M.D., C.M., D.Sc., Cupar-Fife
1883		* Newcombe, Henry, F.R.C.S.E., 5 Dalrymple Crescent, Edinburgh
1884		* Nicholson, J. Shield, M.A., D.Sc., Professor of Political Economy in the University of Edinburgh, 3 Belford Park 350
1880	C.	* Nicol, W. W. J., M.A., D.Sc., 15 Blacket Place

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Date of Election.			
1878		Norris, Richard, M.D., M.R.C.S. Eng., 67 Broad Street, Birmingham	
1888		* Ogilvie, F. Grant, M.A., B.Sc., Principal of the Heriot-Watt College	
1888		* Oliphant, James, M.A., 11 Ramsay Gardens	
1886	C.	Oliver, James, M.D., F.L.S., Physician to the London Hospital for Women, 18 Gordon Square, London	355
1895		Oliver, Thomas, M.D., F.R.C.P., Professor of Physiology in the University of Durham, 7 Ellison Place, Newcastle-upon-Tyne	
1884	K. C.	* Omond, R. Traill, Superintendent of Ben Nevis Observatory, Fort-William, Inverness, and 43 Charlotte Square, Edinburgh	
1877		Panton, George A., 73 Westfield Road, Edgbaston, Birmingham	
1892		Parker, Thomas, Memb. Inst. C.E., Newbridge House, Wolverhampton	
1886	C.	* Paton, D. Noël, M.D., B.Sc., F.R.C.P.E., 33 George Square	360
1889		* Patrick, David, M.A., LL.D., c/o W. & R. Chambers, 339 High Street	
1892		* Paulin, David, 6 Forres Street	
1881	N. C.	* Peach, B. N., F.R.S., F.G.S., Acting Palæontologist of the Geological Survey of Scotland, 86 Findhorn Place	
1889		* Peck, William, F.R.A.S., Town's Astronomer, Murrayfield, Edinburgh	
1863		Peddie, Alexander, M.D., F.R.C.P.E., 15 Rutland Street	365
1887	C.	* Peddie, Wm., D.Sc., Assistant to the Professor of Natural Philosophy, Edinburgh University, 2 Cameron Park	
1886		* Peebles, D. Bruce, Tay House, Bonnington, Edinburgh	
1893		Perkin, Arthur George, 8 Montpellier Terrace, Hyde Park, Leeds	
1889		* Philip, R. W., M.A., M.D., F.R.C.P.E., 4 Melville Crescent	
1883		Phillips, Charles D. F., M.D., LL.D., 10 Henrietta St., Cavendish Sq., London, W.	370
1859	C.	Playfair, The Right Hon. Lord, G.C.B., LL.D., F.R.S., 68 Onslow Gardens, London	
1877	C.	Pole, William, Memb. Inst. C.E., Mus. Doc., F.R.S., Athenæum Club, London	
1886		* Pollock, Charles Frederick, M.D., F.R.C.S.E., 1 Buckingham Terrace, Hillhead, Glasgow	
1874		Powell, Baden Henry Baden-, C.I.E., M.R.A.S., Forest Department, India	
1852		Powell, Eyre B., C.S.I., M.A., 28 Park Road, Haverstock Hill, Hampstead, London	375
1888		Prain, David, Surgeon, Indian Medical Service, and Curator of the Herbarium, Royal Botanic Gardens, Shibpur, Calcutta	
1892		* Pressland, Arthur, M.A., Camb., Edinburgh Academy	
1875	C.	Prevost, E. W., Ph.D., Elton, Newnham, Gloucester	
1849		Primrose, Hon. B. F., C.B., 22 Moray Place	
1885		* Pullar, J. F., Rosebank, Perth	380
1880		* Pullar, Sir Robert, Tayside, Perth	
1884		Ramsay, E. Peirson, M.R.I.A., F.L.S., C.M.Z.S., F.R.G.S., F.G.S., Fellow of the Imperial and Royal Zoological and Botanical Society of Vienna, Curator of Australian Museum, Sydney, N.S.W.	
1891		* Rankine, John, M.A., LL.D., Advocate, Professor of the Law of Scotland in the University of Edinburgh, 23 Ainslie Place	
1885	C.	* Rattray, John, M.A., B.Sc., Dunkeld	
1883	C.	* Readman, J. B., D.Sc., F.C.S., 4 Lindsay Place, Edinburgh	385
1889		Redwood, Boverton, F.I.C., F.C.S., Assoc. Inst. C.E., Glenwathen, Ballard's Lane, Finchley, Middlesex	
1875		* Richardson, Ralph, W.S., 10 Magdala Place	
1872		Ricarde-Seaver, Major F. Ignacio, Athenæum Club, Pall Mall, London	

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Date of Election.			
1883		* Ritchie, R. Peel, M.D., F.R.C.P.E., President of the Scottish Microscopical Society, 1 Melville Crescent	
1880		Roberts, D. Lloyd, M.D., F.R.C.P.L., 23 St John Street, Manchester	390
1872		Robertson, D. M. C. L. Argyll, M.D., LL.D., F.R.C.S.E., Surgeon Oculist to the Queen for Scotland, 18 Charlotte Square	
1886		* Robertson, Right Hon. J. P. B., Q.C., LL.D., Lord Justice-General of Scotland and Lord President of the Court of Session, 19 Drumsheugh Gardens	
1896		* Robertson, Robert, M.A., 27 Hartington Place, Viewforth	
1896	C.	* Robertson, W. G. Aitchison, D.Sc., M.D., F.R.C.P.E., 26 Minto Street	
1877	C.	* Robinson, George Carr, F.I.C., F.C.S., Lecturer on Chemistry in the College of Chemistry, Royal Institution, Hull	395
1881		* Rogerson, John Johnston, B.A., LL.B., LL.D., Merchiston Castle Academy	
1881		Rosebery, The Right Hon. the Earl of, K.G., K.T., LL.D., Dalmeny Park, Edinburgh	
1880		Rowland, L. L., M.A., M.D., President of the Oregon State Medical Society, and Professor of Physiology and Microscopy in Williamette University, Salem, Oregon	
1880		* Russell, Sir James Alex., M.A., B.Sc., M.B., F.R.C.P.E., LL.D., Woodville, Canaan Lane	
1869	C.	Rutherford, Wm., M.D., F.R.C.P.E., F.R.S., Professor of Physiology in the University of Edinburgh, 14 Douglas Crescent	400
1864		Sandford, The Right Rev. Bishop D. F., LL.D., Boldon Rectory, Newcastle-on-Tyne	
1895		Savage, Thomas, M.D., F.R.C.S. England, M.R.C.P. London, Professor of Gynæcology, Mason College, Birmingham, The Ards, Knowle, Warwickshire	
1891		Sawyer, Sir James, Knt., M.D., F.R.C.P., J.P., Consulting Physician to the Queen's Hospital, Haseley Hall, Warwick	
1885	C.	Scott, Alexander, M.A., D.Sc., St Peter's College, Cambridge	
1880		Scott, J. H., M.B., C.M., M.R.C.S., Prof. of Anatomy in the University of Otago, N.Z.	405
1888		* Scott, John, C.B., Memb. Inst. C.E., Halkshill, Largs, Ayrshire	
1875		Scott, Michael, Memb. Inst. C.E., care of A. Grahame, Esq., 30 Great George Street, Westminster	
1889		* Scougal, Andrew E., M.A., H.M. Inspector of Schools, 12 Blantyre Terrace	
1872	C.	Seton, George, M.A., Advocate, Ayton House, Abernethy, Perthshire	
1894		* Shield, Wm., M.Inst.C.E., Executive Engineer, National Harbour of Refuge, Peterhead	410
1872		Sibbald, John, M.D., Comr. in Lunacy, 18 Great King Street	
1870		Sime, James, M.A., Craigmount House, 52 Dick Place	
1871		Simpson, A. R., M.D., President of the Royal College of Physicians, Professor of Midwifery in the University of Edinburgh, 52 Queen Street	
1888		* Sinclair, D. S., 370 Great Western Road, Glasgow	
1876		* Skinner, William, W.S., 35 George Square	415
1868		Smith, Adam Gillies, C.A., Agsacre, North Berwick	
1891	C.	* Smith, Alex., B.Sc., Ph.D., Prof. of General Chemistry, University of Chicago, Ills., U.S.	
1882	C.	Smith, C. Michie, B.Sc., F.R.A.S., Professor of Physical Science, Christian College, and Officiating Government Astronomer, Madras, India	
1885		* Smith, George, F.C.S., Polmont Station	
1883		Smith, James Greig, M.A., M.B., 16 Victoria Square, Clifton	420
1871	C.	Smith, John, M.D., F.R.C.S.E., LL.D., 11 Wemyss Place	
1880		Smith, William Robert, M.D., D.Sc., Barrister-at-Law, Professor of Forensic Medicine in King's College, 74 Great Russell Street, Bloomsbury Square, London	

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Date of Election.			
1846	K. B. C.	Smyth, Piazzi, LL.D., Ex-Astronomer-Royal for Scotland, and Emeritus Professor of Astronomy in the University of Edinburgh, Clova, Ripon	
1880		Sollas, W. J., M.A., D.Sc., F.R.S., late Fellow of St John's College, Cambridge, and Professor of Geology and Mineralogy in the University of Dublin, Lisnabin, Dartry Park Road, Rathgar, county Dublin	
1889	C.	* Somerville, William, Dr Oec., B.Sc., Professor of Agriculture and Forestry in the Durham College of Science, Newcastle-upon-Tyne	425
1882		* Sorley, James, F.I.A., C.A., 32 Onslow Square, London	
1896		* Spence, Frank, M.A., B.Sc., 76 Marchmont Crescent	
1874	C.	* Sprague, T. B., M.A., LL.D., Actuary, 29 Buckingham Terrace	
1891		* Stanfield, Richard, Professor of Mechanics and Engineering in the Heriot-Watt College	
1886	C.	* Stevenson, Charles A., B.Sc., Memb. Inst. C.E., 28 Douglas Crescent	430
1884		* Stevenson, David Alan, B.Sc., Memb. Inst. C.E., 45 Melville Street	
1877		* Stevenson, James, F.R.G.S., Largs	
1888		* Stevenson, Rev. John, LL.D., Minister of Glamis, Forfarshire	
1868		Stevenson, John J., 4 Porchester Gardens, London	
1888	C.	* Stewart, Charles Hunter, D.Sc., M.B., C.M., 3 Carlton Terrace	435
1868		Stewart, Major-General J. H. M. Shaw, late R.E., Assoc. Inst. C.E., F.R.G.S., 61 Lancaster Gate, London, W.	
1866	C.	Stewart, Sir Thomas Grainger, M.D. Edin. and Dub., F.R.C.P.E., LL.D., Professor of the Practice of Physic in the University of Edinburgh, 19 Charlotte Square	
1873		* Stewart, Walter, 1 Murrayfield Gardens	
1877		* Stirling, William, D.Sc., M.D., Brackenbury Professor of Physiology and Histology in Owens College and Victoria University, Manchester	
1889	C.	* Stockman, Ralph, M.D., F.R.C.P.E., 12 Hope Street	440
1894		* Struthers, John, M.D., LL.D., Emeritus Professor of Anatomy in the University of Aberdeen, 24 Buckingham Terrace	
1823		Stuart, Captain T. D., H.M.I.S.	
1896		* Sutherland, John Francis, M.D., Deputy Com. in Lunacy for Scotland, 4 Merchiston Bank Avenue	
1875		* Syme, James, 10 George Street	
1885	C.	* Symington, Johnson, M.D., F.R.C.S.E., Prof. of Anatomy in Queen's College, Belfast	445
1872		Tait, The Venerable A., D.D., LL.D., M.R.I.A., Archdeacon of Tuam, Moylough Rectory, Ballinasloe, County Galway, Ireland	
1861	K. C. V. J.	Tait, P. Guthrie, M.A., D.Sc., Professor of Natural Philosophy in the University of Edinburgh (General Secretary), 38 George Square	
1895		Talmage, James Edward, Ph.D., D.Sc., Pres. of the Univ. of Utah, Salt Lake City, Utah	
1890	C.	Tanakadate, Aikitu, Professor of Natural Philosophy in the Imperial University of Japan, Tokyo, Japan	
1870		Tatlock, Robert R., F.C.S., City Analyst's Office, 156 Bath Street, Glasgow	450
1892		* Taylor, W. A., M.A. (Camb.), 3 East Mayfield	
1872		Teape, Rev. Charles R., M.A., Ph.D., 15 Findhorn Place	
1892		* Thackwell, J. B., M.B., C.M., Tenterfield, New South Wales	
1885	C.	* Thompson, D'Arcy W., B.A., F.L.S., Professor of Natural History in University College, Dundee	
1884		* Thoms, George Hunter, of Aberlemno, Advocate, Sheriff of the Counties of Orkney and Zetland, 13 Charlotte Square	455

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Date of Election.			
1870		Thomson, Rev. Andrew, D.D., 63 Northumberland Street	
1887		* Thomson, Andrew, M.A., D.Sc., Mathematical Master in the Perth Academy, 10 Pitcullen Terrace, Perth	
1887	C.	* Thomson, J. Arthur, M.A., Lecturer on Zoology, School of Medicine, 11 Ramsay Gardens	
1880		Thomson, John Millar, F.R.S., 85 Addison Road, London	
1896		* Thomson, George Ritchie, M.B., C.M., 306 Bath Street, Glasgow	460
1870		Thomson, Spencer C., Actuary, 10 Eglinton Crescent	
1882		Thomson, Wm., M.A., B.Sc., Registrar, University of the Cape of Good Hope, University Chambers, Cape Town	
1876		Thomson, William, Royal Institution, Manchester	
1893		* Tillie, Joseph, M.D., C.M., 10 Castle Terrace	
1874	N. C.	* Traquair, R. H., M.D., LL.D., F.R.S., F.G.S., Keeper of the Natural History Collection in the Museum of Science and Art, Edinburgh, 8 Dean Park Crescent	465
1874		* Tuke, J. Batty, M.D., F.R.C.P.E., 20 Charlotte Square	
1888		* Turnbull, Andrew H., Actuary, The Elms, Whitehouse Loan	
1861	N. C.	Turner, Sir William, M.B., LL.D., D.C.L., D.Sc. Dub., F.R.C.S.E., F.R.S., Professor of Anatomy in the University of Edinburgh, 6 Eton Terrace	
1895		* Turton, Albert H., F.C.S., F.R.G.S., Ashleigh, Carlyle Road, Edgbaston, Birmingham	
1877		* Underhill, Charles E., B.A., M.B., F.R.C.P.E., F.R.C.S.E., 8 Coates Crescent	470
1889		Underhill, T. Edgar, M.D., F.R.C.S.E., Broomsgrove, Worcestershire	
1891		Vernon, Henry Hannotte, M.D., 7 Talbot Street, Southport, Lancashire	
1875		Vincent, Charles Wilson, F.I.C., F.C.S., M.R.I., Librarian of the Reform Club, Pall Mall, 38 Queen's Road, South Hornsey, Middlesex	
1888		Walker, James, Memb. Inst. C.E., Engineer's Office, Harbour Works, Douglas, Isle of Man	
1891	B. C.	* Walker, James, D.Sc., Ph.D., Professor of Chemistry in University College, Dundee, 8 Windsor Terrace, Dundee	475
1873	C.	* Walker, Robert, M.A., University, Aberdeen	
1886	C.	* Wallace, R., F.L.S., Prof. of Agriculture and Rural Economy in the Univ. of Edinburgh	
1891		* Walmsley, R. Mullineux, D.Sc., Principal of the Northampton Institute, Clerkenwell, London	
1866		Watson, Patrick Heron, M.D., F.R.C.S.E., LL.D., 16 Charlotte Square	
1862	C.	Watson, Rev. Robert Boog, B.A., LL.D., F.L.S., President of the Conchological Society, Free Church Manse, Cardross, Dumbartonshire	480
1887		* Webster, H. A., Librarian to the University of Edinburgh, 3 John Street, Portobello	
1896		* Webster, John Clarence, B.A., M.D., F.R.C.P.E., 287 Mountain Street, Montreal, Canada	
1882		* Wenley, James A., Treasurer of the Bank of Scotland, 5 Drumsheugh Gardens	
1896		Wenley, R. M., M.A., D.Sc., D.Phil., Professor of Philosophy in the University of Michigan, U.S.	
1896	C.	White, Philip J., M.B., Prof. of Zoology in University College, Bangor, North Wales	485
1890		White, Sir William Henry, K.C.B., LL.D., F.R.S., Memb. Inst. C.E., Assistant Controller of the Navy, and Director of Naval Construction, The Admiralty, London	
1881		Whitehead, Walter, F.R.C.S.E., Professor of Clinical Surgery, Owens College and Victoria University, 499 Oxford Road, Manchester	
1894		Whymper, Edward, 29 Ludgate Hill, London	
1883		Wickham, R. H. B., M.D., F.R.C.S.E., Medical Superintendent, City and County Lunatic Asylum, Newcastle-on-Tyne, West Mead, Dawlish, South Devon	
1879		* Will, John Charles Ogilvie, M.D., 379 Union Street, Aberdeen	490

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Date of Election.			
1868		Williams, W., Principal and Professor of Veterinary Medicine and Surgery, New Veterinary College, Leith Walk	
1888		* Williamson, George, F.A.S. Scot., 37 Newton Street, Finnart, Greenock	
1879		* Wilson, Andrew, Ph.D., F.L.S., Lecturer on Zoology and Comparative Anatomy, 110 Gilmore Place	
1895		Wilson-Barker, David, F.R.G.S., F.R. Met. Soc., Captain-Superintendent Thames Nautical Training College, H.M.S. "Worcester," Greenhithe, Kent	
1878	C.	* Wilson, Rev. John, M.A., 23 Buccleuch Place	495
1882		Wilson, George, M.A., M.D., 7 Avon Place, Warwick	
1891		* Wilson, John Hardie, D.Sc., The Yorkshire College, Leeds	
1889		Wilson, Robert, Memb. Inst. C.E., St Stephen's Club, and 7 Westminster Chambers, Victoria Street, London	
1870		Winzer, John, Chief Surveyor, Civil Service, Ceylon, Southfield, Paignton, S. Devon	
1886	C.	* Woodhead, German Sims, M.D., F.R.C.P.E., Director of the Laboratories of the Royal Colleges of Physicians (Lond.) and Surgeons (Eng.), Examination Hall, Victoria Embankment, W.C., and 1 Nightingale Lane, Balham, London, S.W.	500
1884		Woods, G. A., M.R.C.S., 16 Adelaide Road, Leamington	
1890		* Wright, Johnstone Christie, care of J. Barker Duncan, 6 Hill Street	
1896		* Wright, Robert Patrick, Professor of Agriculture, West of Scotland Technical College, Glasgow, Laventille, Crow Road, Partick, Glasgow	
1882		* Young, Frank W., F.C.S., Lecturer on Natural Science, High School, Dundee, Woodmuir Park, West Newport, Fife	
1892	C.	Young, George, Ph.D., Firth College, Sheffield	
1896	C.	* Young, James Buchanan, M.B., D.Sc., 35 Montague Street	
1882		* Young, Thomas Graham, Westfield, West Calder	507

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LIST OF HONORARY FELLOWS

AT MARCH 1897.

His Royal Highness The PRINCE OF WALES.

FOREIGNERS (LIMITED TO THIRTY-SIX BY LAW X.).

Elected.		
1897	Alexander Agassiz,	*Cambridge (Mass.).*
1897	E.-H. Amagat,	*Paris.*
1889	Marcellin Pierre Eugène Berthelot,	*Paris.*
1895	Ludwig Boltzmann,	*Vienna.*
1864	Robert Wilhelm Bunsen,	*Heidelberg.*
1897	Stanislao Cannizzaro,	*Rome.*
1883	Luigi Cremona,	*Rome.*
1877	Carl Gegenbaur,	*Heidelberg.*
1888	Ernst Haeckel,	*Jena.*
1883	Julius Hann,	*Vienna.*
1884	Charles Hermite,	*Paris.*
1879	Jules Janssen,	*Paris.*
1864	Albert von Kölliker,	*Würzburg.*
1864	Rudolph Leuckart,	*Leipzig.*
1897	Gabriel Lippmann,	*Paris.*
1895	Éleuthère-Élie-Nicolas Mascart,	*Paris.*
1888	Demetrius Ivanovich Mendeléef,	*St Petersburg.*
1895	Carl Menger,	*Vienna.*
1886	Alphonse Milne-Edwards	*Paris.*
1864	Theodore Mommsen,	*Berlin.*
1897	Fridtjof Nansen,	*Christiania.*
1881	Simon Newcomb,	*Washington.*
1895	Max von Pettenkofer,	*Munich.*
1895	Jules Henri Poincaré,	*Paris.*
1889	Georg Hermann Quincke,	*Heidelberg.*
1886	Alphonse Renard,	*Ghent.*
1897	Ferdinand von Richthofen,	*Berlin.*
1897	Henry A. Rowland,	*Baltimore.*
1897	Giovanni V. Schiaparelli,	*Milan.*
1881	Johannes Iapetus Smith Steenstrup,	*Copenhagen.*
1878	Otto Wilhelm Struve,	*St Petersburg.*
1886	Tobias Robert Thalén,	*Upsala.*
1874	Otto Torell,	*Lund.*
1868	Rudolph Virchow,	*Berlin.*
1892	Gustav Wiedemann,	*Leipzig.*
1897	Ferdinand Zirkel,	*Leipzig.*

Total, 36.

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BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW X.).

Elected.		
1889	**Sir Robert Stawell Ball, Kt., LL.D., F.R.S., M.R.I.A., Lowndean, Professor of Astronomy in the University of Cambridge,**	***Cambridge***
1897	**The Very Rev. John Caird, D.D., LL.D., Principal of the University of Glasgow,**	***Glasgow.***
1892	**Colonel Alexander Ross Clarke, C.B., R.E., F.R.S.,**	***Redhill, Surrey***
1897	**George Howard Darwin, M.A., LL.D., F.R.S., Plumian Professor of Astronomy in the University of Cambridge,**	***Cambridge.***
1895	**Sir J. William Dawson, C.M.G., LL.D., F.R.S.,**	***Montreal.***
1897	**Sir William Flower, K.C.B., LL.D., D.C.L., F.R.S., Director of the Natural History Department, British Museum,**	***London.***
1884	**Edward Frankland, D.C.L., LL.D., F.R.S., Corresp. Mem. Inst. of France,**	***London.***
1892	David Gill, LL.D., F.R.S., Her Majesty's Astronomer at the Cape of Good Hope,	*Cape of Good Hope.*
1895	Albert C. L. G. Günther, Ph.D., F.R.S.,	*London.*
1883	Sir Joseph Dalton Hooker, K.C.S.I., M.D., LL.D., D.C.L., F.R.S., Corresp. Mem. Inst. of France,	*London.*
1884	William Huggins, LL.D., D.C.L., F.R.S., Corresp. Mem. Inst. of France,	*London.*
1892	Sir James Paget, Bart., LL.D., D.C.L., F.R.S., Corresp. Mem. Inst. of France,	*London.*
1886	The Lord Rayleigh, D.C.L., LL.D., D.Sc. Dub., Sec. R.S., Corresp. Mem. Inst. of France,	*London.*
1881	The Rev. George Salmon, D.D., LL.D., D.C.L., F.R.S., Corresp. Mem. Inst. of France,	*Dublin.*
1884	J. S. Burdon Sanderson, M.D., LL.D., D.Sc. Dub., F.R.S.,	*Oxford.*
1864	Sir George Gabriel Stokes, Bart., M.P., LL.D., D.C.L., F.R.S., Corresp. Mem. Inst. of France,	*Cambridge.*
1892	The Right Rev. W. Stubbs, D.D., LL.D., Bishop of Oxford,	*Oxford.*
1874	James Joseph Sylvester, LL.D., F.R.S., Corresp. Mem. Inst. of France,	*Oxford.*
1895	Sir Charles Todd, K.C.M.G., F.R.S., Government Astronomer, South Australia,	*Adelaide.*
1883	Alexander William Williamson, LL.D., F.R.S., Corresp. Mem. Inst. of France,	*London.*

Total, 20.

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ORDINARY FELLOWS ELECTED

During Session 1894–95.

Arranged according to the Date of their Election.

3rd December 1894.

Robert Gervase Alford, M. Inst. C.E. James Edward Talmage, D.Sc., Ph.D

7th January 1895.

Charles Bright, Assoc. M. Inst. C.E. Albert H. Turton, F.C.S.

4th February 1895.

The Most Hon. The Marquis of Lothian, K.T. John Macintyre, M.D.
Surgeon-Major Henry Halcro Johnston, D.Sc., M.D.

4th March 1895.

Professor Thomas Oliver, M.D., F.R.C.P.

1st April 1895.

David Deuchar, F.I.A., F.F.A. James Napier, M.A.
George Sandeman, M.A.

3rd June 1895.

Professor Thomas Savage, M.D., F.R.C.S. Captain David Wilson-Barker, F.R. Met. Soc.

1st July 1895.

Edwin H. Barton, D.Sc.

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FELLOWS DECEASED OR RESIGNED

During Session 1894–95.

ORDINARY FELLOWS DECEASED.

Dr T. A. G. Balfour, F.R.C.P.E.
Professor John S. Blackie.
Benjamin Carrington, M.D. (died 18th Jan. 1893).
Hugh F. C. Cleghorn of Stravithie, M.D., LL.D.
Peter Denny, LL.D., Memb. Inst. C.E.
Patrick Dudgeon of Cargen.
The Right Hon. Lord Moncreiff of Tulliebole, LL.D. (Honorary Vice-President).
Alexander Goodman More, M.R.I.A.
John Shand, M.D., F.R.C.P.E.
Murray Thomson, M.D.
The Rev. Thomas Hardie Turnbull.

RESIGNED.

Dr A. S. Cumming.
George M. Cunningham.
R. J. Harvey Gibson.
John S. Yeo.

HONORARY FELLOWS DECEASED

Session 1894–95.

FORIEGN.

James D. Dana.
Ludvig Sven Lovén.
Louis Pasteur.

BRITISH.

Professor Cayley.
Professor T. H. Huxley.

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ORDINARY FELLOWS ELECTED

During Session 1895-96.

Arranged according to the Date of their Election

6th January 1896.

J. O. Affleck, M.D., F.R.C.P.E.
Prof. D. Campbell Black, M.D., F.F.P.S. Glas.
Andrew J. Herbertson.
Professor J. P. Kuenen, Ph.D.
Robert Robertson, M.A.
John Francis Sutherland, M.D.
John Clarence Webster, B.A., M.D., F.R.C.P.E.
Professor Robert Patrick Wright.

3rd February 1896.

W. J. Aitchison Robertson, D.Sc., M.D., F.R.C.P.E.

2nd March 1896.

Professor Matthew Charteris, M.D.
Frank Spence, M.A.
Professor Philip J. White, M.B.

6th April 1896.

J. W. Butters, M.A., B.Sc.
William Donaldson, M.A.
John Fraser, M.B., F.R.C.P.E.
James M'Lintock, M.D., B.Sc.

4th May 1896.

Professor Francis Gibson Baily, M.A.
Alexander Morgan, M.A., B.Sc.
George Ritchie Thomson, M.B., C.M.
James Buchanan Young, M.B., D.Sc.

1st June 1896.

David Harris.
David Fraser Harris, B.Sc., M.B., C.M.
J. Fletcher Horne, M.D., F.R.C.S.E.
E. F. de Jong, L.R.C.P. and S., Ed., F.R.C.V.S.

6th July 1896.

Alfred A. Murray, M.A., LL.B.
Professor R. M. Wenley, M.A., D.Sc., D.Phil.

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FELLOWS DECEASED OR RESIGNED

DURING SESSION 1895–96.

ORDINARY FELLOWS DECEASED.

JAMES ABERNETHY, Memb. Inst. C.E.
Sir THOMAS DAWSON BRODIE, Bart.
The Rev. J. GIBSON CAZENOVE, M.A., D.D.
DAVID CUNNINGHAM, Memb. Inst. C.E.
JOHN GRIEVE, M.A., M.D.
ROBERT LAWSON, M.D.
JOSIAH LIVINGSTON.
GEORGE M'ROBERTS, F.C.S.
HUGH MILLER.
JAMES CLERK RATTRAY, M.D.
Rev. THOMAS MILVILLE RAVEN, M.A.
GEORGE ROBERTSON, Memb. Inst. C.E.
ROBERT MACFIE THORBURN.

RESIGNED.

GEORGE SANDEMAN.
Sir W. J. BELL.
Professor PERKIN.

HONORARY FELLOWS DECEASED.

SESSION 1895–96.

FOREIGN.

ERNST CURTIUS.
GABRIEL AUGUSTE DAUBRÉE.
ARMAND HIPPOLYTÉ LOUIS FIZEAU.
AUGUST KEKULÉ.
HUBERT A. NEWTON.

BRITISH.

Sir WILLIAM ROBERT GROVE.
JOHN RUSSELL HIND.

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LAWS

OF THE

ROYAL SOCIETY OF EDINBURGH,

AS REVISED 20TH FEBRUARY 1882.

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LAWS.

[By the Charter of the Society (printed in the *Transactions*, Vol. VI. p. 5), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

I.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Honorary Fellows. Title.

II.

Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter.* The fees of Ordinary Fellows residing in Scotland.

III.

All Fellows who shall have paid Twenty-five years' annual contribution shall be exempted from further payment. Payment to cease after 25 years.

IV.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt Fees of Non-Resident Ordinary Fellows.

* A modification of this rule, in certain cases, was agreed to at a Meeting of the Society held on the 3rd January 1831.

At the Meeting of the Society, on the 5th January 1857, when the reduction of the Contributions from £3, 3s. to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future annual Contributions.

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Case of Fellows becoming Non-Resident.

from any further payment. In the case of any Resident Fellow ceasing to reside in Scotland, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.

V.

Defaulters.

Members failing to pay their contributions for three successive years (due application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

VI.

Privileges of Ordinary Fellows.

None but Ordinary Fellows shall bear any office in the Society, or vote in the choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Society.

VII.

Numbers Unlimited.

The number of Ordinary Fellows shall be unlimited.

VIII.

Fellows entitled to Transactions.

The Ordinary Fellows, upon producing an order from the TREASURER, shall be entitled to receive from the Publisher, gratis, the Parts of the Society's Transactions which shall be published subsequent to their admission.

IX.

Mode of Recommending Ordinary Fellows.

Candidates for admission as Ordinary Fellows shall make an application in writing, and shall produce along with it a certificate of recommendation to the purport below,* signed by at least *four* Ordinary Fellows, two of whom shall certify their recommendation from personal knowledge. This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall afterwards be printed in the circulars for three Ordinary Meetings of the Society, previous to the day of election, and shall lie upon the table during that time.

* "A. B., a gentleman well versed in Science (*or Polite Literature, as the case may be*), being "to our knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, we hereby "recommend him as deserving of that honour, and as likely to prove a useful and valuable Member."

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X.

Honorary Fellows shall not be subject to any contribution. This class shall consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states. **Honorary Fellows, British and Foreign.**

XI.

Personages of Royal Blood may be elected Honorary Fellows, without regard to the limitation of numbers specified in Law X. **Royal Personages.**

XII.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one meeting, and printed in the circulars for two ordinary meetings of the Society, previous to the day of election. **Recommendation of Honorary Fellows.** **Mode of Election.**

XIII.

The election of Ordinary Fellows shall only take place at the first Ordinary Meeting of each month during the Session. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote. **Election of Ordinary Fellows.**

XIV.

The Ordinary Meetings shall be held on the first and third Mondays of every month from December to July inclusively; excepting when there are five Mondays in January, in which case the Meetings for that month shall be held on its third and fifth Mondays. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make. **Ordinary Meetings.**

* We hereby recommend_____ for the distinction of being made an Honorary Fellow of this Society, declaring that each of us from our own knowledge of his services to (*Literature or Science, as the case may be*) believe him to be worthy of that honour.

(To be signed by three Ordinary Fellows.)

_____

_____

_____

To the President and Council of the Royal Society of Edinburgh.

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XV.

The Transactions.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which they shall deem it expedient to publish in the *Transactions* of the Society, and shall superintend the printing of the same.

The Council shall have power to regulate the private business of the Society. At any Meeting of the Council the Chairman shall have a casting as well as a deliberative vote.

XVI.

How Published.

The Transactions shall be published in parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society.

XVII.

The Council.

That there shall be formed a Council, consisting—First, of such gentlemen as may have filled the office of President; and Secondly, of the following to be annually elected, viz.:—a President, Six Vice-Presidents (two at least of whom shall be resident), Twelve Ordinary Fellows as Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

XVIII.

Retiring Councillors.

Four Councillors shall go out annually, to be taken according to the order in which they stand on the list of the Council.

XIX.

Election of Office-Bearers.

An Extraordinary Meeting for the Election of Office-Bearers shall be held on the fourth Monday of November annually.

XX.

Special Meetings; how called.

Special Meetings of the Society may be called by the Secretary, by direction of the Council; or on a requisition signed by six or more Ordinary Fellows. Notice of not less than two days must be given of such Meetings.

XXI.

Treasurer's Duties.

The Treasurer shall receive and disburse the money belonging to the Society, granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the Extraordinary Meeting in November, he shall present the accounts for the preceding year, duly audited.

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At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

XXII.

At the Extraordinary Meeting in November, a professional accountant shall be chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions. **Auditor.**

XXIII.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes he shall, when necessary, employ a clerk, to be paid by the Society. **General Secretary's Duties.**

XXIV.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the proceedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the Donors' names, to the Curator of the Library and Museum; they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty. **Secretaries to Ordinary Meetings.**

XXV.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the Hall, for the inspection of the Fellows. **Curator of Museum and Library.**

XXVI.

All Articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations, as the Council from time to time shall appoint. **Use of Museum and Library.**

XXVII.

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date. **Register Book.**

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THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND GUNNING VICTORIA JUBILEE PRIZES.

The above Prizes will be awarded by the Council in the following manner :—

I. KEITH PRIZE.

The KEITH PRIZE, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1897–98 for the "best communication on a scientific subject, communicated, in the first instance, to the Royal Society during the Sessions 1895–96 and 1896–97." Preference will be given to a paper containing a discovery.

II. MAKDOUGALL-BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded at the commencement of the Session 1898–99, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.

2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 8th July 1898.

3. The Competition is open to all men of science.

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4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets, superscribed with the same motto, and containing the name of the Author.

5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society. They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the paper shall be published in the Transactions.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented to the Society during the Sessions 1896–97, 1897–98, whether they may have been given in with a view to the prize or not.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr PATRICK NEILL of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate,

1. The NEILL PRIZE, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1898–99.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented to the Society during the three years preceding the 8th July 1898,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

IV. GUNNING VICTORIA JUBILEE PRIZE.

This Prize, founded in the year 1887 by Dr R. H. GUNNING, is to be awarded triennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

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Evidence of such work may be afforded either by a Paper presented to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize consists of a sum of money, and is open to men of science resident in or connected with Scotland. The first award was made in the year 1887.

In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

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AWARDS OF THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND GUNNING VICTORIA JUBILEE PRIZES, FROM 1827 TO 1896.

I. KEITH PRIZE.

1st Biennial Period, 1827–29.—Dr Brewster, for his papers "on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals," published in the Transactions of the Society.

2nd Biennial Period, 1829–31.—Dr Brewster, for his paper "on a New Analysis of Solar Light," published in the Transactions of the Society.

3rd Biennial Period, 1831–33.—Thomas Graham, Esq., for his paper "on the Law of the Diffusion of Gases," published in the Transactions of the Society.

4th Biennial Period, 1833–35.—Professor J. D. Forbes, for his paper "on the Refraction and Polarization of Heat," published in the Transactions of the Society.

5th Biennial Period, 1835–37.—John Scott Russell, Esq., for his Researches "on Hydrodynamics," published in the Transactions of the Society.

6th Biennial Period, 1837–39.—Mr John Shaw, for his experiments "on the Development and Growth of the Salmon," published in the Transactions of the Society.

7th Biennial Period, 1839–41.—Not awarded.

8th Biennial Period, 1841–43.—Professor James David Forbes, for his papers "on Glaciers," published in the Proceedings of the Society.

9th Biennial Period, 1843–45.—Not awarded.

10th Biennial Period, 1845–47.—General Sir Thomas Brisbane, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.

11th Biennial Period, 1847–49.—Not awarded.

12th Biennial Period, 1849–51.—Professor Kelland, for his papers "on General Differentiation, including his more recent communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations," published in the Transactions of the Society.

13th Biennial Period, 1851–53.—W. J. Macquorn Rankine, Esq., for his series of papers "on the Mechanical Action of Heat," published in the Transactions of the Society.

14th Biennial Period, 1853–55.—Dr Thomas Anderson, for his papers "on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances," published in the Transactions of the Society.

15th Biennial Period, 1855–57.—Professor Boole, for his Memoir "on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments," published in the Transactions of the Society.

16th Biennial Period, 1857–59.—Not awarded.

17th Biennial Period, 1859–61.—John Allan Broun, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers "on the Horizontal Force of the Earth's Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally," published in the Transactions of the Society.

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18TH BIENNIAL PERIOD, 1861–63.—Professor WILLIAM THOMSON, of the University of Glasgow, for his Communication "on some Kinematical and Dynamical Theorems."

19TH BIENNIAL PERIOD, 1863–65.—Principal FORBES, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars," published in the Transactions of the Society.

20TH BIENNIAL PERIOD, 1865–67.—Professor C. PIAZZI SMYTH, for his paper "on Recent Measures at the Great Pyramid," published in the Transactions of the Society.

21ST BIENNIAL PERIOD, 1867–69.—Professor P. G. TAIT, for his paper "on the Rotation of a Rigid Body about a Fixed Point," published in the Transactions of the Society.

22ND BIENNIAL PERIOD, 1869–71.—Professor CLERK MAXWELL, for his paper "on Figures, Frames, and Diagrams of Forces," published in the Transactions of the Society.

23RD BIENNIAL PERIOD, 1871–73.—Professor P. G. TAIT, for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.

24TH BIENNIAL PERIOD, 1873–75.—Professor CRUM BROWN, for his Researches "on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."

25TH BIENNIAL PERIOD, 1875–77.—Professor M. FORSTER HEDDLE, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.

26TH BIENNIAL PERIOD, 1877–79.—Professor H. C. FLEEMING JENKIN, for his paper "on the Application of Graphic Methods to the Determination of the Efficiency of Machinery," published in the Transactions of the Society; Part II. having appeared in the volume for 1877–78.

27TH BIENNIAL PERIOD, 1879–81.—Professor GEORGE CHRYSTAL, for his paper "on the Differential Telephone," published in the Transactions of the Society.

28TH BIENNIAL PERIOD, 1881–83.—THOMAS MUIR, Esq., LL.D., for his "Researches into the Theory of Determinants and Continued Fractions," published in the Proceedings of the Society.

29TH BIENNIAL PERIOD, 1883–85.—JOHN AITKEN, Esq., for his paper "on the Formation of Small Clear Spaces in Dusty Air," and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society.

30TH BIENNIAL PERIOD, 1885–87.—JOHN YOUNG BUCHANAN, Esq., for a series of communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, &c.; two of which, viz., "On Ice and Brines," and "On the Distribution of Temperature in the Antarctic Ocean," have been published in the Proceedings of the Society.

31ST BIENNIAL PERIOD, 1887–89.—Professor E. A. LETTS, for his Papers on the Organic Compounds of Phosphorus, published in the Transactions of the Society.

32ND BIENNIAL PERIOD, 1889–91.—R. T. OMOND, Esq., for his Contributions to Meteorological Science, many of which are contained in Vol. XXXIV. of the Society's Transactions.

33RD BIENNIAL PERIOD, 1891–93.—Professor THOMAS R. FRASER, F.R.S., for his Papers on *Strophanthus hispidus*, Strophanthin, and Strophanthidin, read to the Society in February and June 1889 and in December 1891, and printed in Vols. XXXV., XXXVI., and XXXVII. of the Society's Transactions.

34TH BIENNIAL PERIOD, 1893–95.—Dr CARGILL G. KNOTT, for his papers on the Strains produced by Magnetism in Iron and in Nickel, which have appeared in the Transactions and Proceedings of the Society.

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II. MAKDOUGALL-BRISBANE PRIZE.

1st Biennial Period, 1859.—Sir Roderick Impey Murchison, on account of his Contributions to the Geology of Scotland.

2nd Biennial Period, 1860–62.—William Seller, M.D., F.R.C.P.E., for his "Memoir of the Life and Writings of Dr Robert Whytt," published in the Transactions of the Society.

3rd Biennial Period, 1862–64.—John Denis Macdonald, Esq., R.N., F.R.S., Surgeon of H.M.S. "Icarus," for his paper "on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology," published in the Transactions of the Society.

4th Biennial Period, 1864–66.—Not awarded.

5th Biennial Period, 1866–68.—Dr Alexander Crum Brown and Dr Thomas Richard Fraser, for their conjoint paper "on the Connection between Chemical Constitution and Physiological Action," published in the Transactions of the Society.

6th Biennial Period, 1868–70.—Not awarded.

7th Biennial Period, 1870–72.—George James Allman, M.D., F.R.S., Emeritus Professor of Natural History, for his paper "on the Homological Relations of the Cœlenterata," published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.

8th Biennial Period, 1872–74.—Professor Lister, for his paper "on the Germ Theory of Putrefaction and the Fermentive Changes," communicated to the Society, 7th April 1873.

9th Biennial Period, 1874–76.—Alexander Buchan, A.M., for his paper "on the Diurnal Oscillation of the Barometer," published in the Transactions of the Society.

10th Biennial Period, 1876–78.—Professor Archibald Geikie, for his paper "on the Old Red Sandstone of Western Europe," published in the Transactions of the Society.

11th Biennial Period, 1878–80.—Professor Piazzi Smyth, Astronomer-Royal for Scotland, for his paper "on the Solar Spectrum in 1877–78, with some Practical Idea of its probable Temperature of Origination," published in the Transactions of the Society.

12th Biennial Period, 1880–82.—Professor James Geikie, for his "Contributions to the Geology of the North-West of Europe," including his paper "on the Geology of the Faroes," published in the Transactions of the Society.

13th Biennial Period, 1882–84.—Edward Sang, Esq., LL.D., for his paper "on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor," and generally for his Recalculation of Logarithms both of Numbers and Trigonometrical Ratios, —the former communication being published in the Proceedings of the Society.

14th Biennial Period, 1884–86.—John Murray, Esq., LL.D., for his papers "On the Drainage Areas of Continents, and Ocean Deposits," "The Rainfall of the Globe, and Discharge of Rivers," "The Height of the Land and Depth of the Ocean," and "The Distribution of Temperature in the Scottish Lochs as affected by the Wind."

15th Biennial Period, 1886–88.—Archibald Geikie, Esq., LL.D., for numerous communications, especially that entitled "History of Volcanic Action during the Tertiary Period in the British Isles," published in the Transactions of the Society.

16th Biennial Period, 1888–90.—Dr Ludwig Becker, for his Paper on "The Solar Spectrum at Medium and Low Altitudes," printed in Vol. XXXVI. Part I. of the Society's Transactions.

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17TH BIENNIAL PERIOD, 1890–92.—HUGH ROBERT MILL, Esq., D.Sc., for his Papers on "The Physical Conditions of the Clyde Sea Area," Part I. being already published in Vol. XXXVI. of the Society's Transactions.

18TH BIENNIAL PERIOD, 1892–94.—Professor JAMES WALKER, D.Sc., Ph.D., for his work on Physical Chemistry, part of which has been published in the Proceedings of the Society, Vol. XX., pp. 255–263. In making this award, the Council took into consideration the work done by Professor Walker along with Professor Crum Brown on the Electrolytic Synthesis of Dibasic Acids, published in the Transactions of the Society.

19TH BIENNIAL PERIOD, 1894–96.—Professor JOHN G. M'KENDRICK, for numerous Physiological papers, especially in connection with Sound; many of which have appeared in the Society's publications.

III. THE NEILL PRIZE.

1ST TRIENNIAL PERIOD, 1856–59.—Dr W. LAUDER LINDSAY, for his paper "on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliaceous Lichens," published in the Transactions of the Society.

2ND TRIENNIAL PERIOD, 1859–62.—ROBERT KAYE GREVILLE, LL.D., for his Contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.

3RD TRIENNIAL PERIOD, 1862–65.—ANDREW CROMBIE RAMSAY, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and Memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geographical Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

4TH TRIENNIAL PERIOD, 1865–68.—Dr WILLIAM CARMICHAEL M'INTOSH, for his paper "on the Structure of the British Nemerteans, and on some New British Annelids," published in the Transactions of the Society.

5TH TRIENNIAL PERIOD, 1868–71.—Professor WILLIAM TURNER, for his papers "on the great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Fœtal Membranes in the Cetacea," published in the Transactions of the Society.

6TH TRIENNIAL PERIOD 1871–74.—CHARLES WILLIAM PEACH, Esq., for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

7TH TRIENNIAL PERIOD, 1874–77.—Dr RAMSAY H. TRAQUAIR, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton), published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.

8TH TRIENNIAL PERIOD, 1877–80.—JOHN MURRAY, Esq., for his paper "on the Structure and Origin of Coral Reefs and Islands," published (in abstract) in the Proceedings of the Society.

9TH TRIENNIAL PERIOD, 1880–83.—Professor HERDMAN, for his papers "on the Tunicata," published in the Proceedings and Transactions of the Society.

10TH TRIENNIAL PERIOD, 1883–86.—B. N. PEACH, Esq., for his Contributions to the Geology and Palæontology of Scotland, published in the Transactions of the Society.

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11TH TRIENNIAL PERIOD, 1886–89.—ROBERT KIDSTON, Esq., for his Researches in Fossil Botany, published in the Transactions of the Society.

12TH TRIENNIAL PERIOD, 1889–92.—JOHN HORNE, Esq., F.G.S., for his Investigations into the Geological Structure and Petrology of the North-West Highlands.

13TH TRIENNIAL PERIOD, 1892–95.—ROBERT IRVINE, Esq., for his papers on the action of Organisms in the Secretion of Carbonate of Lime and Silica, and on the solution of these substances in Organic Juices. These are printed in the Society's Transactions and Proceedings.

IV. GUNNING VICTORIA JUBILEE PRIZE.

1ST TRIENNIAL PERIOD, 1884–87.—Sir WILLIAM THOMSON, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices, which have been communicated to the Society.

2ND TRIENNIAL PERIOD, 1887–90.—Professor P. G. TAIT, Sec. R.S.E., for his work in connection with the "Challenger" Expedition, and his other Researches in Physical Science.

3RD TRIENNIAL PERIOD, 1890–93.—ALEXANDER BUCHAN, Esq., LL.D., for his varied, extensive, and extremely important Contributions to Meteorology, many of which have appeared in the Society's Publications.

4TH TRIENNIAL PERIOD, 1893–96.—JOHN AITKEN, Esq., for his brilliant Investigations in Physics, especially in connection with the Formation and Condensation of Aqueous Vapour.

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PROCEEDINGS

OF THE

STATUTORY GENERAL MEETINGS,

26TH NOVEMBER 1894,

AND

25TH NOVEMBER 1895.

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STATUTORY MEETING.

HUNDRED AND TWELFTH SESSION.

Monday, 26th November 1894.

At the Annual Statutory Meeting,

Sir DOUGLAS MACLAGAN, M.D., President, in the Chair,

The Minutes of last Annual Statutory Meeting of 27th November 1893 were read, approved, and signed.

Drs SMITH and BALFOUR were requested to act as Scrutineers in the Ballot for the New Council.

Read Letter of Resignation by the TREASURER, and Letter of Council in reply.

On the motion of the CHAIRMAN, seconded by Dr BUCHAN, a cordial vote of thanks was passed to Dr GILLIES SMITH for his many services to the Society.

The Annual Accounts were submitted and approved.

The Auditor's Report was read and approved.

The Scrutineers reported that the following Council had been duly elected:—

Sir DOUGLAS MACLAGAN, M.D., F.R.C.P.E., President.
Sir WM. TURNER, LL.D., D.C.L., F.R.S.,
Professor RALPH COPELAND, Ph.D., Astronomer-Royal for Scotland,
Professor JAMES GEIKIE, LL.D., D.C.L., F.R.S.,
The Hon. Lord MACLAREN, LL.D., F.R.A.S.,
The Rev. Professor FLINT, D.D.,
Professor JOHN G. M'KENDRICK, M.D., LL.D., F.R.S.,
} Vice-Presidents.
Professor P. G. TAIT, M.A., D.Sc., General Secretary.
Professor CRUM BROWN, F.R.S.,
JOHN MURRAY, LL.D.,
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PHILIP R. D. MACLAGAN, F.F.A., Treasurer.
ALEXANDER BUCHAN, M.A., LL.D., Curator of Library and Museum.

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COUNCILLORS.

Professor D'Arcy W. Thompson, B.A., F.L.S.	A. Beatson Bell, Advocate.
Professor J. Shield Nicholson, M.A., D.Sc.	Sir Arthur Mitchell, K.C.B., LL.D.
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Alexander Bruce, M.A., M.D.	D. Noël Paton, B.Sc., M.D.
Professor Fred. O. Bower, M.A., F.R.S.	Cargill G. Knott, D.Sc.

The Scrutineers were thanked for their services by the Chairman.

On the motion of Mr Cox, the Auditor was thanked and reappointed.

On the motion of Professor Tait, the Chairman was thanked for his conduct in the chair.

Douglas Maclagan, *P.*

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STATUTORY MEETING.

HUNDRED AND THIRTEENTH SESSION.

Monday, 25th November 1895.

At the Annual Statutory Meeting,

Sir DOUGLAS MACLAGAN, M.D., President, in the Chair,

The Minutes of last Annual Statutory Meeting of 26th November 1894 were read, approved, and signed.

Mr SKINNER and Mr OMOND were appointed Scrutineers, and the Ballot for the New Council commenced.

The TREASURER submitted his Annual Accounts for the year with the Auditors' Report, which were read and approved.

The Scrutineers reported that the following New Council had been duly elected:—

The Right Hon. Lord KELVIN, LL.D., D.C.L., F.R.S., President.

Professor RALPH COPELAND, Astronomer-Royal for Scotland,
Professor JAMES GEIKIE, LL.D., F.R.S.,
The Hon. Lord M'LAREN, LL.D., F.R.A.S.,
The Rev. Professor FLINT, D.D.,
Professor JOHN G. M'KENDRICK, M.D., LL.D., F.R.S.,
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} Vice-Presidents.

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Professor FREDERICK O. BOWER, M.A., F.R.S.
A. BEATSON BELL, Advocate.
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CARGILL G. KNOTT, D.Sc.
Sir WILLIAM TURNER, LL.D., D.C.L., F.R.S.
Sir STAIR AGNEW, K.C.B., M.A.
JAMES BURGESS, C.I.E., LL.D., M.R.A.S.
JOHN STURGEON MACKAY, M.A., LL.D.

On the motion of Mr SKINNER, seconded by Dr CRUM BROWN, the thanks of the Society were given to the Treasurer.

Dr CRUM BROWN proposed, seconded by Dr MUNRO, a vote of thanks to the Scrutineers.

Mr BELL moved, and Professor TAIT seconded, a special vote to the retiring President (the Chairman), who in returning thanks called attention to the award of a Royal Medal to Dr JOHN MURRAY.

The AUDITOR was thanked for his Report and reappointed.

A. CAMPBELL FRASER, D.C.L.,
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VOL. XXXVIII.

EDINBURGH:
PUBLISHED BY ROBERT GRANT & SON, 107 PRINCES STREET,
AND WILLIAMS & NORGATE, 14 HENRIETTA STREET, COVENT GARDEN, LONDON.

MDCCCXCVII.

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No.	I.	Published	November 13, 1894.	No.	XIII.	Published	April 21, 1896.
„	II.	„	May 5, 1895.	„	XIV.	„	June 12, 1896.
„	III.	„	August 3, 1895.	„	XV.	„	July 17, 1896.
„	IV.	„	September 20, 1895.	„	XVI.	„	September 11, 1896.
„	V.	„	February 5, 1896.	„	XVII.	„	September 10, 1896.
„	VI.	„	October 10, 1895.	„	XVIII.	„	August 12, 1896.
„	VII.	„	December 7, 1895.	„	XIX.	„	September 1, 1896.
„	VIII.	„	December 9, 1895.	„	XX.	„	October 12, 1896.
„	IX.	„	January 15, 1896.	„	XXI.	„	October 19, 1896.
„	X.	„	January 10, 1896.	„	XXII.	„	November 25, 1896.
„	XI.	„	February 4, 1896.	„	XXIII.	„	October 26, 1896.
„	XII.	„	February 2, 1896.	„	XXIV.	„	November 16, 1896.

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